Drainage equation

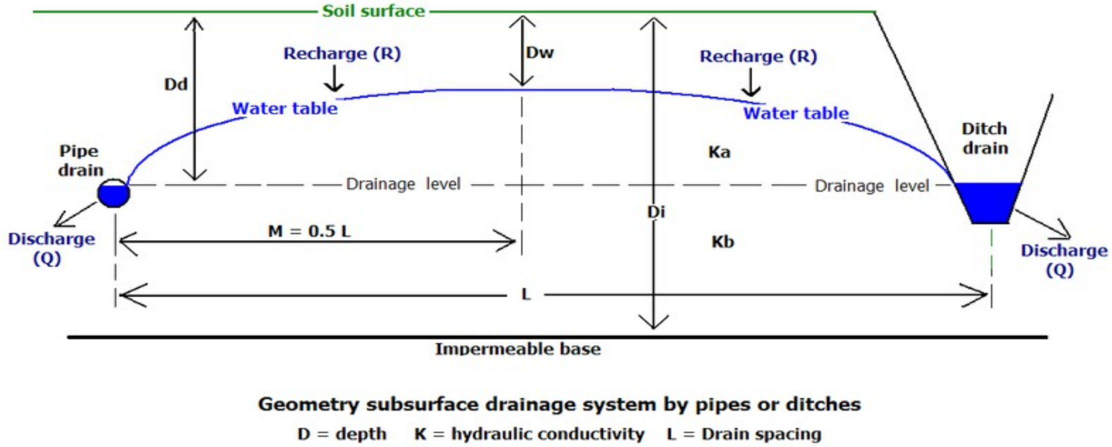
On website https://www.waterlog.info/

A **drainage equation** is an equation describing the relation between depth and spacing of parallel subsurface drains, of the watertable, depth and hydraulic conductivity of the soils. It is used in drainage design.

The equation is used to design a subsurface drainage system to solve the problem of elevated water tables, to improve the agricultural conditions and the crop yields.

Contents

1. Hooghoudt's equation



Parameters in Hooghoudt's drainage equation

A well known steady-state drainage equation is the Hooghoudt drain spacing equation. Its original publication is in Dutch.The equation was introduced in the USA by van Schilfgaarde.

$$Q\,L^2 = 8\,Kb\,De\,(Di - Dd)\,(Dd - Dw) + 4\,Ka\,(Dd - Dw)^2$$

where:

- Q = steady state drainage discharge rate (m/day)
- Ka = hydraulic conductivity of the soil above drain level (m/day)
- Kb = hydraulic conductivity of the soil below drain level (m/day)
- Di = depth of the impermeable layer below drain level (m)
- Dd = depth of the drains (m)
- Dw = steady state depth of the watertable midway between the drains (m)
- L = spacing between the drains (m)
- De= equivalent depth, a function of L, (Di-Dd), and r
- r = drain radius (m)

The equivalent depth De represents the reduction of the thickness of the aquifer (Di-Dd) to simulate the effect of the risistance to the radial flow towards the drainage ditch or the pipe drain

Steady (equilibrium) state condition
In steady-state, the level of the water table remains constant and the discharge rate (Q) equals the rate of groundwater recharge (R), i.e. the amount of water entering the groundwater through the watertable per unit of time. By considering a long-term (e.g. seasonal) average depth of the water table (Dw) in combination with the long-term average recharge rate (R), the net storage of water in that period of time is negligibly small and the steady state condition is satisfied: one obtains a dynamic equilibrium.

Considering a sufficiently long time span (for example a saeson of the year), the change of the amount of water stored at the water table is normally negligible small compared to the total amount of water drained in that period, so that the condition of steady-state is almost exactly approached and the steady-state drainage equation is applicable.

Derivation of the equation

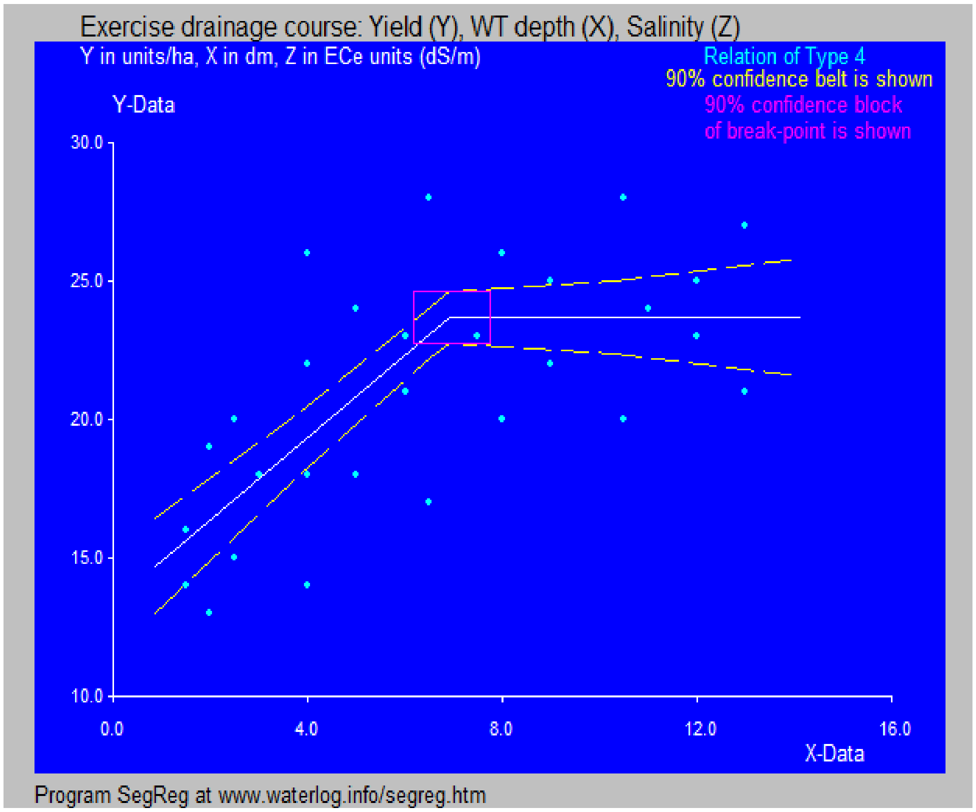
For the derivation of the equation Hooghoudt used the law of Darcy, the summation of circular potential functions and, for the determination of the influence of the impermeable layer, de method of mirror images and superposition.

Hooghoudt published tables for the determination of the equivalent depth (d), because the function (F) in De = F (L,Di-Dd,r) consists of long series of terms.

Determining:

- the discharge rate (Q) from the recharge rate (R) in a water balance as detailed in the article: hydrology (agriculture)
- the permissible long term average depth of the water table (Dw) permitted for the plants on the basis of *agricultural drainage criteria*
- the soil's hydraulic conductivity (Ka and Kb) by measurements
- the depth of the bottom of the aquifer (Di)

the design drain spacing (L) can be found from the equation in dependence of the drain depth (Dd) and drain radius (r).



Crop yield and seasonal average depth of the water table

2. Equivalent depth

In 1991 a closed-form expression was developed for the equivalent depth (d) that can replace the Hooghoudt tables (W.H. van der Molen en J.Wesseling, 1991) :

$$De = \pi L / 8 \; \{ \ln(L / \pi r) + F(x) \}$$

wherw:

$$x = 2\pi \, (Di - Dd) / L$$
$$F(x) = \Sigma \; 4e^{-2nx} / n \, (1 - e^{-2nx})$$

with:

$n = 1, 3, 5, \ldots$

e = 2.71 . . . , the number e, basis of the Niperian logarithm.

3. AMPLIFICATION

Definitions of drainage of sloping land and entrance resistance

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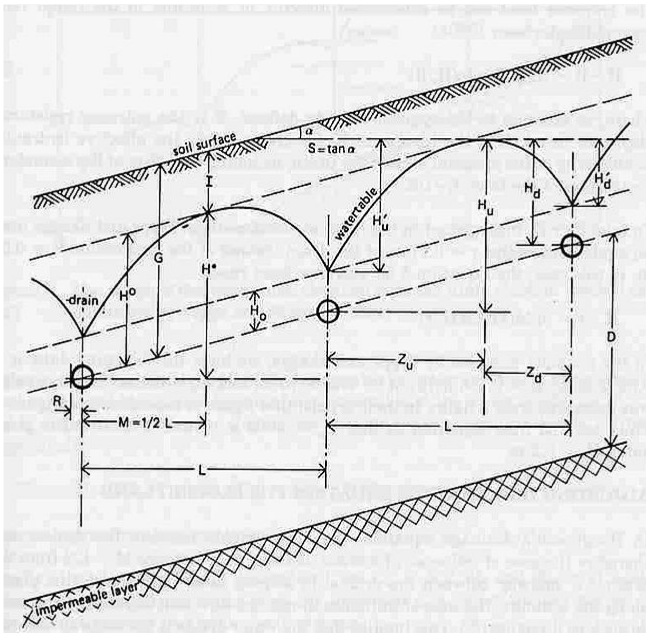

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<td>Geometry used in the theory of subsurface drainage in sloping land.</td>
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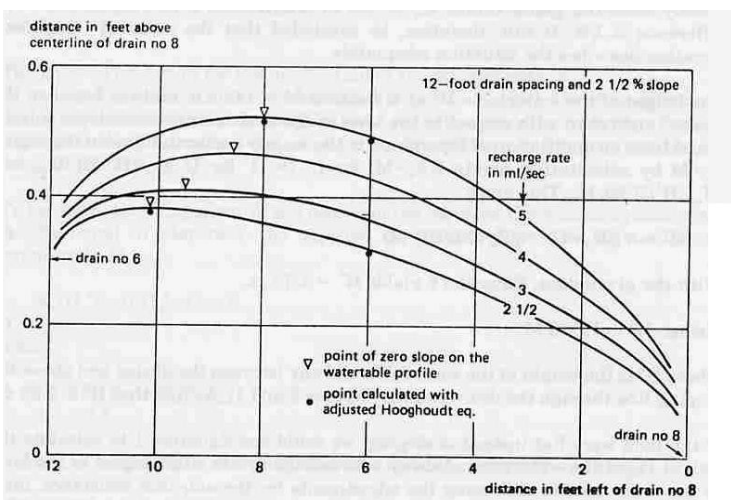

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<td>Corroboration with experiments in sloping sand tanks.</td>
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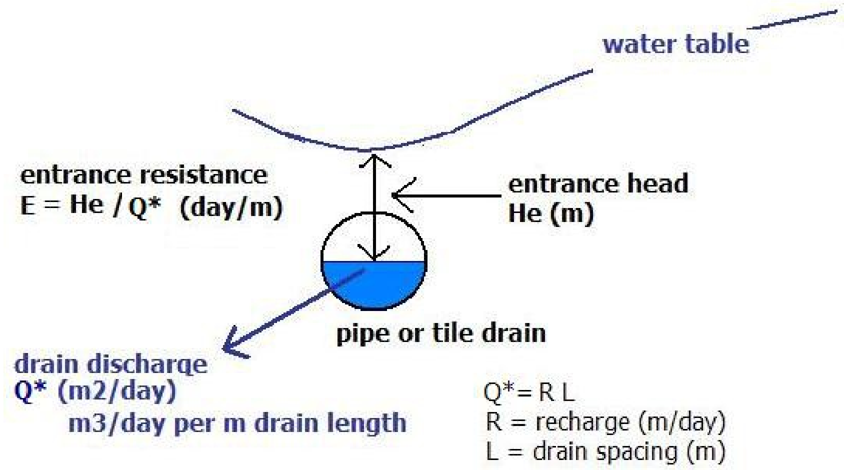
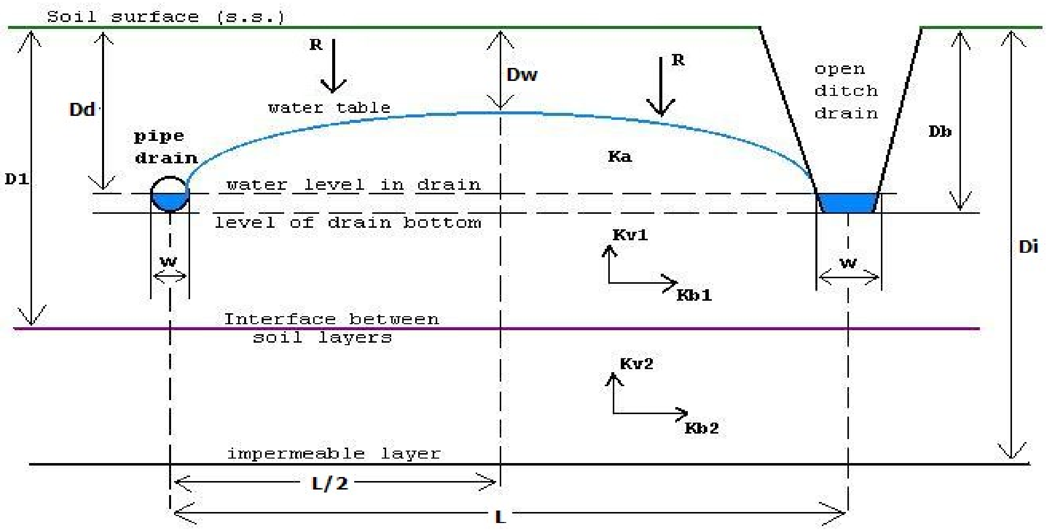
	Definition of the entrance resistance

Theoretically, Hooghoudt's equation can also be used for *sloping land*. The theory on drainage of sloping land is corroborated by the results of sand tank experiments (Zeigler 1972.) In addition, the *entrance resistance* encountered by the water upon entering the drains can be accounted for.

4. Extended use

The drainage formula can be amplified to account for (see figure below):

- the additional energy associated with the incoming percolation water (recharge), see groundwater energy balance
- multiple soil layers
- Anisotropic hydraulic conductivity, the vertical conductivity (Kv) being different from the horizontal (Kh)
- drains of different dimensions with any width (W)
- entrance resistance



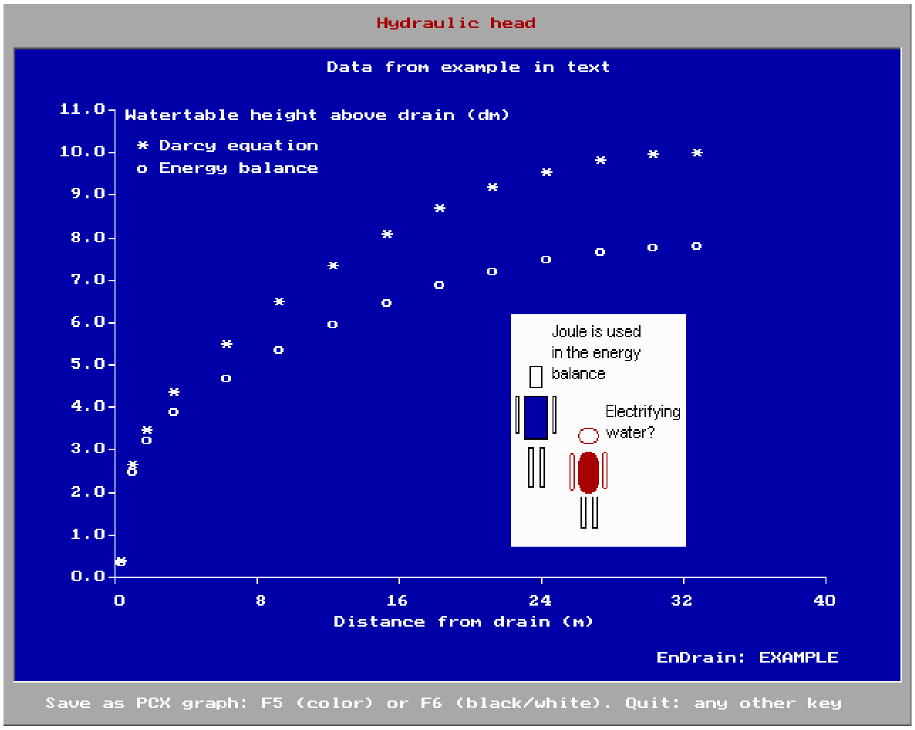
Amplification of the parameters used by Hooghoudt

The amplified drainage equation uses an hydraulic equivalent of Joule's law in electricity.

It is in the form of a differential equation that cannot be solved analytically (i.e. in a closed form) but the solution requires a numerical method for which a computer program is indispensable.

The availability of a computer program also helps in quickly assessing various alternatives and performing a sensitivity analysis.

The blue figure below shows an example of results of a computer aided calculation with the amplified drainage equation using the EnDrain program. It shows that incorporation of the incoming energy associated with the recharge leads to a somewhat deeper water table.



EnDrain program: drainage and the shape of the water table

Alternative

The possibility exists to simulate the movement of the watertable in the course of the time under influence of a varying recharge by rainfall or irrigation.

Existe la posibilidad de simular el movimiento de la napa freática en el transcurso del tiempo bajo la influencia de un recarga (por lluvia o riego) variable.

See: https://www.waterlog.info/rainoff.htm

5. References

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