THE ENERGY BALANCE OF GROUNDWATER FLOW APPLIED TO DITCH DRAINAGE IN LAYERED AND/OR ANISOTROPIC SOILS

On: https://www.waterlog.info/endrain.htm

1. Introduction, ditch drains

Agricultural land drainage is widely practiced in the world, but it needs to be applied with care [Ref. 1]. Crop yields are reduced when the water table is too shallow (Ref. 2, figure 1).
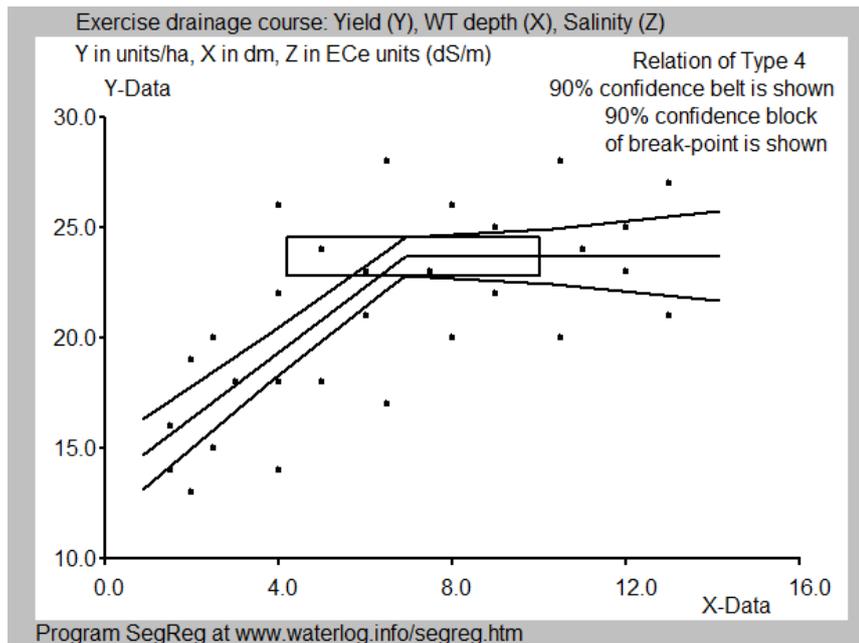
Fig. 1 Statistical analysis of the relation between crop yield (Y) and seasonal average depth of the water table (X in dm). When the water table is shallower than 8 dm the yields decline.

The depth of the water table depends on the drain distance (spacing). Spacing calculations between consecutive lateral drains are closely related to water flow towards the drains [Ref. 3].

Boonstra and Rao [Ref. 4], therefore, introduced the complete energy balance of groundwater flow. It is based on equating the change of hydraulic energy flux over a horizontal distance to the conversion rate of hydraulic energy into to friction of flow over that distance. The energy flux is calculated on the basis of a multiplication of the hydraulic potential and the flow velocity, integrated over the total flow depth. The conversion rate is determined in analogy to the heat loss equation of an electric current, named after Joule. For the differentiation of the integral equation, Leibniz's integral rule had to be applied.

Assuming (1) steady state fluxes, i.e. no water and associated energy is stored, (2) vertically two-dimensional flow, i.e. the flow pattern repeats itself in parallel vertical planes, (3) the horizontal component of the flow is constant in a vertical cross-section, and (4) the soil's hydraulic conductivity is constant from place to place, it was found that [Ref. 4]::

$$\frac{dJ}{dX} = \frac{Vx}{Kx} - \frac{R(J-Jr)}{Vx.J} \qquad \text{(Eq. 1)}$$

where:

J is the level of the water table at distance X, taken with respect to the level of the impermeable base of the aquifer (m), Jr is a reference value of level J (m), X is a distance in horizontal direction (m), Vx is the apparent flow velocity at X in

horizontal   X-direction (m/day), Kx is the horizontal hydraulic conductivity (m/day), R is the steady recharge by downward percolating water stemming from rain or irrigation water (m/day), dX is a small increment of distance X (m), dJ is the increment of level J over increment dX (m), dJ/dX is the gradient of the water table at X (m/m).

The last term of equation 1 represents the energy associated with the recharge R. When the recharge R is zero, Equation 1 yields Darcy's equation, which does not account for the complete energy balance. The negative sign before Vx indicates that the flow is positive when the gradient dJ/dX is negative, i.e. the flow follows the descending gradient, and vice versa.
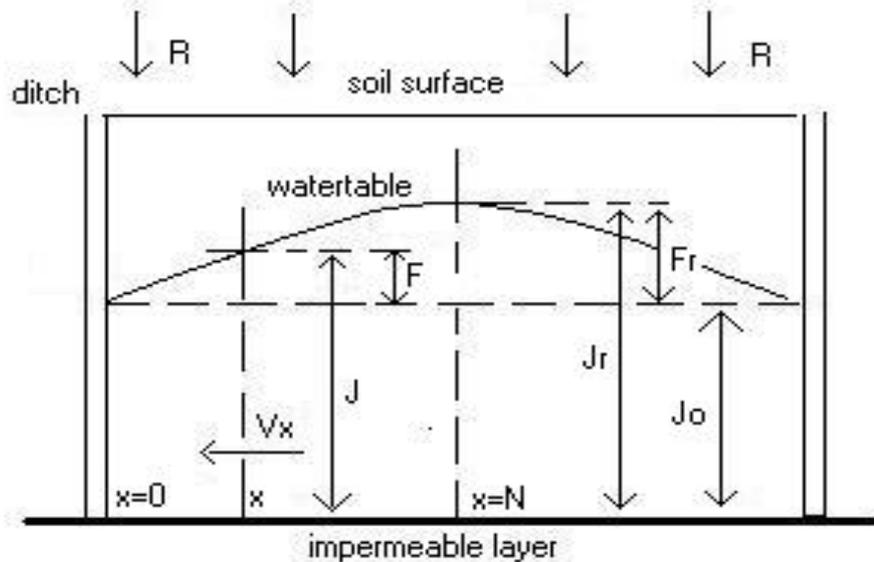
Fig. 2. Vertically two-dimensional flow of ground water to parallel ditches resting on the impermeable base of a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation.

Figure 2 shows the vertically two-dimensional flow of ground water to parallel ditches resting on a horizontal impermeable base of a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation (R>0, m/day). At the distance X=N (m), i.e. midway between the ditches, there is a water divide. Here the water table is horizontal. At the distance X≤N, the discharge of the aquifer equals $Q = -R(N-X)$ (m²/day) where the minus sign indicates that the flow is contrary to the X direction. From this water balance we find $Vx = Q/J = -R(N-X)/J$ (m/day).

With this expression for the velocity Vx, Equation 1 can be changed into:

$$\frac{dJ}{dX} = \frac{R(N-X)}{Kx.J} - \frac{Jr-J}{N-X} \qquad \text{(Eq. 2)}$$

Setting F = J−Jo, and Fr = Jr−J, where Jo is the value of J at X=0, i.e. at the edge of the ditch, it is seen that F represents the level of the water table with respect to the water level in the ditch (the drainage level). Applying the condition that dF/dX=0 at X=N, we find from Equation 2 that Fr=Fn, where Fn is the value of F at X=N, and:

$$\frac{F}{X} = \frac{R(N-X)}{Kx.J} - \frac{Fn-F}{N-X} \qquad \text{(Eq. 3)}$$

Introducing the equivalent drain radius C (m), and integrating equation 3 from X=C to any other value X, gives:

$$F = \int_C^X \frac{R(N-X)}{Kx \cdot J} \, dX - \int_C^X \frac{Fn-F}{N-X} \, dX \qquad \text{(Eq. 4)}$$

Integration of the last term in equation 4 requires advance knowledge of the level Fn. To overcome this problem, a numerical solution and a trial and error procedure must be sought. Oosterbaan et al. [Ref. 4] gave a method of numerical solution and an example from which it was found that the water table is lower than calculated according to the traditional method, except at the place of the ditch.

2. NUMERICAL INTEGRATION

For the numerical integration, the horizontal distance N is divided into a number (T) of equally small elements with length U, so that $U = N / T$. The elements are numbered S = 1, 2, 3, ..., T.

The height F at a distance defined by the largest value of distance X in element S, is denoted as F_S. The change of height F over the S-th element is denoted as G_S, and found from:

$$G_S = F_S - F_{S-1}$$

The average value of height F over the S-th element is:

$$\underline{F}_S = F_{S-1} + \tfrac{1}{2}G_{S-1}$$

For the first step (S=i, see Equation 10 below), the value of $\underline{F}_S = \underline{F}_i$ must be determined by trial and error because then the slope $G_{S-1} = G_{i-1}$ is not known.

The average value of the horizontal distance X of the S-th element is found as:

$$\underline{X}_S = U(S-0.5)$$

The average value of depth Y over the S-th element is:

$$\underline{Y}_S = \tfrac{1}{2}\pi\underline{X}_S \qquad \text{when} \quad C < \underline{X}_S < Xi \qquad \text{(Eq. 5a)}$$

$$\underline{Y}_S = D \qquad \text{when} \quad Xi < \underline{X}_S < N \qquad \text{(Eq. 5b)}$$

Equation 3 can now be approximated by:

$$G_S = U(A_S + B_S) \qquad \text{(Eq. 6)}$$

where:

$$A_S = R(N - \underline{X}_S) / Z_S$$

with:

$$Z_S = Kx(\underline{Y}_S + \underline{F}_S) \qquad \text{when} \quad C < \underline{X}_S < Xi \qquad \text{(Eq. 7a)}$$

$$Z_S = Kx(D + \underline{F}_S) \qquad \text{when} \quad Xi < \underline{X}_S < N \qquad \text{(Eq. 7b)}$$

and:

$$B_S = (\underline{F}_S - F_T) / (N - \underline{X}_S)$$

where F_T is the value of $\underline{F}_S$ when S = T. The factor Z_S can be called transmissivity (m^2/day) of the aquifer.

Now, the height of the water table at any distance X can be found, conform to Equations 6a and 6b, from:

$$F_S = \sum_{i}^{S} G_S \qquad \text{(Eq. 8)}$$

where i is the initial value of the summations, found as the integer value of:

$$i = 1 + C/U \qquad \text{(Eq. 9)}$$

so that the summation starts at the outside of the drain.

Since F_S depends on B_S and B_S on F_S and F_T, which is not known in advance, Equations 8 and 10 must be solved by iterations.

Omitting the last terms of Equations 6a and 6b, i.e. ignoring part of the energy balance, and further in similarity to the above procedure, a value G_S* can be found as:

$$G_S* = R.U(N-\underline{X}_S)/Z_S* \qquad \text{(Eq. 10)}$$

where:

$$Z_S* = Kx(\underline{Y}_S+\underline{F}_S*) \quad \text{when} \quad C<\underline{X}_S<Xi$$

$$Z_S* = Kx(D+\underline{F}_S*) \quad \text{when} \quad Xi<\underline{X}_S<N$$

and:

$$\underline{F}_S* = F_{S-1}*+\tfrac{1}{2}G_{S-1}*$$

Thus the height of the water table, in conformity to Equation 10, is:

$$F_S* = \sum_{i}^{S} G_S* \qquad \text{(Eq. 11)}$$

This equation corresponds to the classical Hooghoudt equation [Ref. 5] and will be used for comparison with Equation 10 (accounting for the energy balance).

3. Example of a numerical solution

When the width of the water body in the ditch (Wd) is twice its depth (Dd), then the principles are exactly the same (the ditches are neutral). Only the radius C of the drain must be replaced by an equivalent radius $Ce = Dd = \tfrac{1}{2}Wd$ (figure 3).
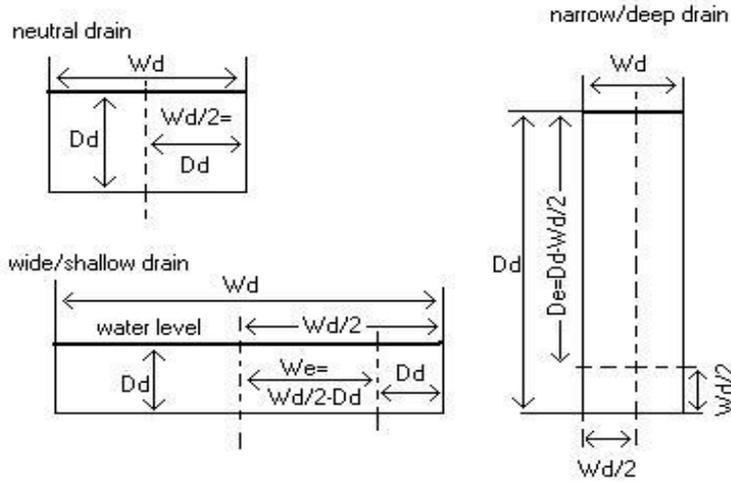
Fig. 3. Vertical and horizontal dimensions of ditch drains.

In conformity to the flow near pipe drains, the water enters the ditch from one side radially over a perimeter $\frac{1}{2}\pi Ce$. The numerical calculations start at the distance $X = \frac{1}{2}Wd$ from the central axis of the ditch. This means that the initial value i (Eq. 11) is changed into the integer value of:

$$i' = 1 + \frac{1}{2}Wd / U \qquad\qquad\qquad \text{(Eq. 12)}$$

The corresponding value of the horizontal distance X is indicated by X_i'.

The depth Y of the sloping impermeable layer is taken with respect to the water level in the drain. Otherwise the calculations are the same as for pipes.

For other situations (figure 3), we distinguish wide ditches ($\frac{1}{2}Wd > Dd$) from narrow ditches ($\frac{1}{2}Wd < Dd$).

For wide ditches, we replace the radius C by an equivalent radius $Cw = Dd$, and we define the excess width as $We = \frac{1}{2}Wd - Dd$. The initial value i is again changed into i' as in equation 14. Further, the value $\underline{Y}_S$ in equation 7a changes into:

$$\underline{Y}_S' = \frac{1}{2}\pi X_S \qquad\qquad [\frac{1}{2}Wd < X_S < Xi'] \qquad \text{(Eq. 13)}$$

and the value of Z_S in Equation 9a changes into:

$$Z_S' = Kx(\underline{F}_S + \underline{Y}_S' + We) \qquad [\frac{1}{2}Wd < X_S < Xi'] \qquad \text{(Eq. 14)}$$

For narrow ditches, the radius C is replaced by an equivalent radius $Cn = \frac{1}{2}Wd$, and we define the excess dept as $De = Dd - \frac{1}{2}Wd$. Like before, the initial value i is changed into i'. Further, the factor Z_S in equation 7a is changed into:

$$Z_S'' = Kx(\underline{F}_S + \underline{Y}_S + De) \qquad [Dd < X_S < Xi'] \qquad \text{(Eq. 15)}$$

An example of results of calculations with the energy balance is given in table 1 for different ditches but otherwise with the same data as in the example for pipe drains. All ditches have a wet surface area of 2 m^2.

The calculations for the numerical solutions were made on a computer with the EnDrain program [Ref. 6].

Table 1. Results of the calculations of the height Fn of the water table, taken with respect to the drainage level, midway between ditches of different shapes, using a numerical and iterative solution of the hydraulic energy balance for the conditions described the example of Section 4, applying Equations 8 and 10 with steps U=0.05 m and making the adjustments as described in Section 5.

Width Wd (m)	Depth Dd (m)	Equivalent radius (m)	Type of Ditch	Elevation of water table (Fn, m)
2	1	1	Neutral	0.55
3	2/3	2/3	Wide – shallow	0.52
1	2	½	Narrow – deep	0.52

4. Anisotropy

The hydraulic conductivity of the soil may change with depth and be different in horizontal and vertical direction [Ref. 7]. We will distinguish a horizontal conductivity Ka of the soil above drainage level, and a horizontal and vertical conductivity Kb and Kv below drainage level. The following principles are only valid when $Kv > R$, otherwise the recharge R percolates downwards only partially and the assumed water balance $Q = -R(N-X)$ is not applicable.

The effect of the conductivity Kv is taken into account by introducing the anisotropy ratio $A = \sqrt{(Kb/Kv)}$, as described by Boumans [Ref. 8].

The conductivity Kb is divided by this ratio, yielding a transformed conductivity: $Kt = Kb/A = \sqrt{(Kb.Kv)}$. As normally $Kv < Kb$, we find $A > 1$ and $Kt < Kb$. On the other hand, the depth of the aquifer below the bottom level of the drain is multiplied with the ratio.

Hence the transformed depth is: $Dt = A.D$.

The distance $X_i = 2D/\pi$ (equation 5) of the radial flow now changes into $Xt = 2Dt/\pi$. When $A > 1$, the transformed distance Xt is larger than Xi. The effect of the transformation is that the extended area of radial flow and the reduced conductivity Kt increase the resistance to the flow and enlarges the height of the water table.

Including the entrance resistance, the transmissivity Z_S (Equations 7a and 7b), for different types of drains, now becomes:

for neutral ditches if *[Ce<$\underline{X_S}$<Xt]*:
$$Z_S = \tfrac{1}{2}\pi Kt.\underline{X_S} + (Kb-Kt)Dd + Ka.\underline{F_S}$$

for wide ditches if *[Cw<$\underline{X_S}$<Xt]*:
$$Z_S = \tfrac{1}{2}\pi Kt.\underline{X_S} + (Kb-Kt)Dd + Kv.We + Ka.\underline{F_S}$$

for narrow ditches if *[Cn<$\underline{X_S}$<Xt]*:
$$Z_S = \tfrac{1}{2}\pi Kt.\underline{X_S} - \tfrac{1}{2}Kt.Wd + Kb.Dd + Ka.\underline{F_S}$$

and:

for all ditches when *[Xt<$\underline{X_S}$<N]*:
$$Z_S = Kt.Dt + Ka.\underline{F_S}$$

The suggestion of Boumans [89] to use the wet perimeter of the ditches to find the equivalent radius, without making a distinction between wide and narrow drains, is not followed as this would lead to erroneous results for narrow and very deep drains, especially when they penetrate to the impermeable layer. In the latter case there is no radial flow but the use of the wet perimeter would introduce it. The proposed method does not.

Table 3 gives an example of energy balance calculations for pipe drains in soils with anisotropic hydraulic conductivity using Ka = Kb = 0.14, as in the previous examples, and Kv = 0.14, 0.014 and 0.0014. This yields anisotropy ratios A = 1, 3.16, and 10 respectively. All other data are the same as in the previous examples.

Table 3. Results of the calculations of the height Fn (m) of the water table, taken with respect to the drainage level , midway between ditches in anisotropic soils with a fixed value of the horizontal hydraulic conductivity Kb=0.14 m/day, using a numerical and iterative solution of the hydraulic energy balance for the conditions described the previous examples, employing Equations 8 and 10 with steps U=0.01 m and making the adjustments as described in Section 7.

Vertical hydraulic conductivity Kv (m/day)	Neutral Wd = 2 m Dd = 1 m	Wide Wd = 3 m Dd = 2/3 m	Narrow Wd = 1 m Dd = 2 m
0.14	0.55	0.52	0.52
0.014	0.69	0.73	0.59
0.0014	1.00	1.11	0.74

The table shows that the height Fn increases with increasing ratio A. The narrow/deep ditches show by far the smallest increase of the height Fn, due to their deeper penetration into the soil by which they make use of the higher horizontal conductivity Kb.

Unfortunately, it is practically very difficult to establish and maintain such deep drains at field level.

When the height Fn would be fixed, one would see that the spacing in anisotropic soils is by far the largest for the narrow and deep ditches. Neutral drains would have smaller spacing than wide drains, i.e. the advantage of wide ditches in isotropic soils vanishes in anisotropic soils. The pipe drains would have the smallest spacing.

5. Layered (an)isotropic soils

The soil may consist of distinct (an)isotropic layers. In the following model, three layers are discerned.

The first layer reaches to a depth D1 below the soil surface, corresponding to the depth Wd of the water level in the drain, and it has an isotropic hydraulic conductivity Ka. The layer represents the soil conditions above drainage level.

The second layer has a reaches to depth D2 below the soil surface (D2>D1). It has horizontal and vertical hydraulic conductivities K2x and K2v respectively with an anisotropy ratio A2.The transformed conductivity is Kt2 = K2x/A2.

The third layer rests on the impermeable base at a depth D3 (D3>D2). It has a thickness T3=D3–D2 and horizontal and vertical hydraulic conductivity Kx3 and Kv3 respectively with an anisotropy ratio A3. The transformed conductivity is Kt3= K3x/A3, and the transformed thickness is Tt3=A3.T3

When the thickness T3 = 0 and/or the conductivity K3 = 0 (i.e. the third layer has zero transmissivity and is an impermeable base), the depth D2 may be both larger or smaller than the bottom depth Db of the drain. Otherwise, the depth D2 must be greater than the sum of bottom depth and the (equivalent) radius (C* = C, Ce, Cw, or Cn) of the drain, lest the radial flow component to the drain is difficult to calculate.

For neutral and wide ditch drains, and with D2 > Dw + C* = Dw + Dd, the transformed thickness of the second soil layer below drainage level becomes:

$$Tt2 = A2(D2−Dw).$$

For narrow ditches we have similarly:

$$Tt2 = A2(D2 − Dw − \tfrac{1}{2}Wd + Dd)$$

With the introduction of an additional soil layer, the expressions of transmissivity $\underline{Z}_S$ in Section 7 need again adjustment, as there may two distances Xt (Xt1 and Xt2) of radial flow instead of one, as the radial flow may occur in the second and the third soil layer:

$$Xt1 = 2T2t/\pi$$
$$Xt2 = Xt1 + 2Tt3/\pi$$

With these boundaries, the transmissivities become

for neutral ditches if $[Ce<\underline{X}_S<Xt1]$:
$$Z_S = \tfrac{1}{2}\pi\, Kt2.\underline{X}_S + (Kx2−Kt2)Dd + Ka.\underline{F}_S$$

for wide ditches if $[Cw<\underline{X}_S<Xt1]$:
$$Z_S = \tfrac{1}{2}\pi\, t2.\underline{X}_S+(Kx2−Kt2)Dd+Kv2.We+Ka.\underline{F}_S$$

for narrow ditches $[Cn<\underline{X}_S<Xt]$:
$$Z_S = \tfrac{1}{2}\pi\, Kt2.\underline{X}_S − \tfrac{1}{2}Kt2.Wd + Kx2.Dd + Ka.\underline{F}_S$$

for all ditches:
   if $[Xt1<\underline{X}_S<Xt2]$:
$$Z_S = Kt2.Tt2 + \tfrac{1}{2}\pi\, Kt3.\underline{X}_S + Ka.\underline{F}_S$$
  if $[\underline{X}_S>Tt2+Tt3]$:
$$Z_S = Kt2.Tt2 + Kt3.Tt3 + Ka.\underline{F}_S$$

An example will be given for pipe drains situated at different depths within the relatively slowly permeable second layer having different anisotropy ratios and being underlain by an isotropic, relatively rapidly permeable, third layer with different conductivities. We have the following data:

N = 38 m   C = 0.05 m   R = 0.007 m/day
D1 = 1.0 m   D2 = 2.0 m   D3 = 6.0 m
              Kx2 = 0.5 m/day Kx3 = 1.0 m/day
Ka = 0.5 m/day Kv2= 0.5 m/day Kv3 =1.0 m/day

and variations:

Kv2 = 0.1 m/day   Kv2 = 0.05 m/day
Kx3 = Kv3 = 2.0 m/day   Kx3 = Kv3 = 5.0 m/day

The results are shown in Table 5.

Table 5. Results of the calculations of the height Fn (m) of the water table, taken with respect to the drainage level, midway between ditches in a layered soil of which the second layer, in which the drains are situated, has varying anisotropy ratios with a fixed value of the horizontal hydraulic conductivity Kx2=0.5 m/day, using a numerical and iterative solution of the hydraulic energy balance for the conditions described the example of Section 8, employing Equations 8 and 10 with steps U=0.05 m and making the adjustments as described in Section 8.

Hydr. Cond. 3rd layer Kx3=Kv3 (m/day)	Vert. Hydr. Cond. 2nd layer Kv2 (m/day)	Anisotropy ratio A2 2nd layer (−)	Height Fn of the water table above drainage level (m)	
1.0	0.5	1.0	0.54	
1.0	0.1	2.24	0.75	
1.0	0.05	3.13		0.86
2.0	0.5	1.0	0.45	
2.0	0.1	2.24	0.67	
2.0	0.05	3.13		0.79
5.0	0.5	1.0	0.37	
5.0	0.1	2.24	0.60	
5.0	0.05	3.13		0.74

These results indicate that both the conductivity of the 3rd layer and the anisotropy of the 2nd layer, in which the drains are situated, exert a considerable influence on the height Fn.

In the Netherlands, it is customary to prescribe a minimum permissible depth of the water table of 0.5 m at a discharge of 7 mm/day, which is exceeded on average only once a year. In the example, with a drain depth of 1.0 m, this condition is fulfilled when the height Fn is at most 0.5 m. Here, this occurs when Kv2 is at least 0.5 m/day and when Kx3 = Kv3 is at least 2.0 m. To meet the prescription in the other cases of the example, either the drain depth should be deeper or the drain spacing narrower.

6. Equivalent hydraulic conductivity

For ditch drains reaching the impermeable layer or pipe drains resting on the impermeable layer, Hooghoudt derived the following equation [Ref. 5]:

$$Q = \{8K_b \, D \, F_r + 4K_a \, F_r^2\} / L^2$$

For the meaning of the symbols see figures 2 and 3. Note that J_0 in figure 2 equals D in figure. 3. Q is the drain discharge in m^3/day per m^2 surface area drained or m/day. This equation is based on the partial energy balance,

With a numerical approach, the discharge-weighted average transmissivity (Z_{av}) can be calculated. It is found by dividing the distance from the drain to the point midway between the drains into small steps and determining in each corresponding vertical section the transmissivity below the water level in the well (in m^2/day) and the discharge (in m^2/day). Next these two quantities are multiplied and the products are added. Finally Z_{av} is obtained by dividing this sum of products by the sum of the discharges.

Using $K_e = Z_{av}/D$, where K_e is the equivalent hydraulic conductivity, one will be able to use the above well flow

equation for fully penetrating drains to a situation with partially penetrating drains not reaching the impermeable layer replacing here K_b by K_e.

7. Conclusions

Application of the complete energy balance of groundwater flow to pipe ditch drains leads to lower elevations of the water table or, if the elevation is fixed, to a wider drain spacing compared to standard formulas

A numerical solution, moreover, can give the shape of the water table. Further, it can take the anisotropy of the soil's hydraulic conductivity into account.

Calculations with the full energy balance need be done on a computer because of the cumbersome iterative, numerical procedure required. EnDrain may be useful software for that purpose. A similar program for drainage by pumped well is also available [Ref. 9].

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