

## DETERMINATION OF FORMAL CONFIDENCE INTERVALS IN CASE OF LINEAR REGRESSION WITH BREAKPOINT (BP)

The two regression equations are:

$$RLa = Aa (X - AvXa) + AvYa$$

$$RLb = Ab (X - AvXb) + AvYb$$

where  $Aa$  is the regression coefficient to the left of BP,  $Ab$  is the regression coefficient to the right of BP,  $X$  is a distance along the horizontal axis,  $AvXa$  is the average of the  $X$  values smaller than BP,  $AvXb$  is the average of the  $X$  values larger than BP,  $AvYa$  is the average of the  $Y$  values of the data with  $X < BP$ , and  $AvYb$  is the average of the  $Y$  values of the data with  $X > BP$ .

The **upper** confidence line to the **left** of BP is found from the relation:

$$(X, Y1a + ts \cdot StDevYc)$$

where:

$$Y1a = Aa (X - AvXa) + AvYa$$

$$StDevYc = \sqrt{ \{ s_{\gamma}^2 + (X - AvXt)^2 \cdot StDevA^2 \} }$$

with:

$$s_{\gamma}^2 = \{ StDevYra^2 (Na-1) + StDevYrb^2 (Nb-1) \} / Nt (Nt-2)$$

and:

$$StDevA^2 = \{ StDevYra^2 (Na-1) + StDevYrb^2 (Nb-1) \} / (Nt-2) RedSumX^2$$

with:

$$RedSumX^2 = (Nt-1)(StDevX)^2$$

where:

$AvXt$  is the average of all  $X$ -data,  $StDevYra$  is the standard deviation of the residuals of  $Y$  values after regression (or of the distances between the  $Y$  values and  $RLa$ ,  $StDevYr$ ) to the left of BP,  $StDevYrb$  is the standard deviation of the residuals of  $Y$  values after regression (or of the distances between the  $Y$  values and  $RLb$ ,  $StDevYr$ ) to the right of BP,  $Na$  is the number of data sets with  $X < BP$ ,  $Nb$  is the number of data sets with  $X > BP$ ,  $Nt$  is the total number of data sets ( $Nt = Na + Nb$ ), and  $StDevX$  is the standard deviation of all the  $X$ -data (i.e. in all data sets).

Similarly, the **lower** confidence line to the **left** of BP is found from the relation:

$$(X, Y1a - ts \cdot StDevYc)$$

The **upper** confidence line to the **right** of BP is found from the relation:

$$(X, Y1b + ts \cdot StDevYc)$$

where:

$$Y1b = Ab (X - AvXb) + AvYb$$

Similarly, the **lower** confidence line to the **right** of BP is found from the relation:

$$(X, Y1b - ts \cdot StDevYc)$$