HOOGHOUDT'S DRAINAGE EQUATION, ADJUSTED FOR ENTRANCE RESISTANCE AND SLOPING LAND

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ABSTRACT

Hooghoudt's drainage equation for flat land is adjusted in this article to cover the drainage of sloping land by horizontal and parallel subsurface pipe drains with entrance resistance. The drains are laid along the contours at equal depth below the soil surface, which receives a steady recharge evenly distributed over the area. The adjusted equations are tested against the results obtained with a numerical method of finite elements described by Fipps and Skaggs (1989), and against the results of sand tank experiments described by Zeigler (1972). The tests show a good agreement. In practice, the adjusted Hooghoudt equation is easier to apply than the numerical and scale models.

1 INTRODUCTION

Recently, Fipps and Skaggs (1989) reviewed published experiments and theories of the subsurface drainage of sloping land, and presented their own analysis based on the method of finite elements. They considered the case of subsurface drainage by parallel and horizontal pipe drains with entrance resistance, placed at equal depth along the contours of the sloping land, which receives a steady recharge evenly distributed over the area.

LeSaffre (1987) mentioned that so far no analytical formulae have been determined to show the relationship between all the parameters involved in the drainage of sloping land. He therefore presented the derivation of such formulae and gave a simple solution for drains resting on an impervious layer. In other cases, his solutions are not explicit, and they do not include the effect of entrance resistance.

In the following, the Hooghoudt equation (Hooghoudt 1940) will be adjusted to enable an explicit and fairly simple mathematical description of the relations between the parameters, including the depth to an impermeable layer, entrance resistance, and the slope of the land.

The adjusted equation will be applied to the same drainage situation as described by Fipps and Skaggs (1989), and the results of the Hooghoudt method will be compared with those obtained by Fipps and Skaggs with the finite element method. In addition, a comparison will be made with the results of sand tank experiments reported by Zeigler (1972).

2 HOOGHOUDT'S DRAINAGE EQUATION

Hooghoudt's drainage equation (Hooghoudt 1940) gives a mathematical relation of the parameters involved in the subsurface drainage of flat land by a system of horizontal and parallel ditches or pipe drains without entrance resistance, placed at equal depth and subject to a steady recharge evenly distributed over the area (Figure 1).

The most widely known form of Hooghoudt's equation was presented by Wesseling (1972). In a slightly modified form, it reads:

$$qL = (8Hm/L)(Kb.De+Ka.Ha) \qquad (1)$$

where q is the steady recharge of water percolating to the water table equal to the drain discharge (m/day or m/hr), L is the drain spacing (m), Hm is the height of the water table mid-way between drains, taken with respect to the centre of the drain (m), Kb is the hydraulic conductivity of the soil below drain level (m/day or m/hr), Ka is the hydraulic conductivity of the soil above drain level (m/day or m/hr), De is Hooghoudt's equivalent depth to the impermeable layer below drain level, and Ha=Hm/2 is the average height of the water table above drain level.

The equivalent depth De depends on the depth D of the impermeable layer below the drains as follows:

If $D<R$:	$De = D$	(2a)
If $R<D<L/4$:	$De = D.L/\{(L-D2)+8D.L.\ln(D/R)\}$	(2b)
If $D>L/4$:	$De = L/8\ln(L/R)$	(2c)

where R is the drain radius (m). For $L/8<D<L/2$, Equations 2b and 2c give almost the same result. Equation 2b is the outcome of an analysis of Hooghoudt's theory by Labeye (1960) as reported by Wesseling (1972). Wesseling, however, gives a different expression for depth De when $R<D<L/4$, which is not based on Hooghoudt's drainage equation, but on that of Ernst. The two expressions yield only a very small difference in the values of De. Equations 2a and 2c were given by Hooghoudt (1940).

If the drains are open ditches instead of buried pipes, the above equations are applicable with an equivalent radius calculated as $R=W/\pi$, where W is the wetted perimeter of the ditch.

The above equations can be adjusted to take an-isotropic hydraulic conductivity into account (Boumans 1979). They can also be adjusted to take the resistance to vertical downward flow into account using the principles described by Oosterbaan (1986). Further, if the coefficient 8 is changed into 6.4, the equations can be used for drainage with a falling water table (Oosterbaan et al. 1989).

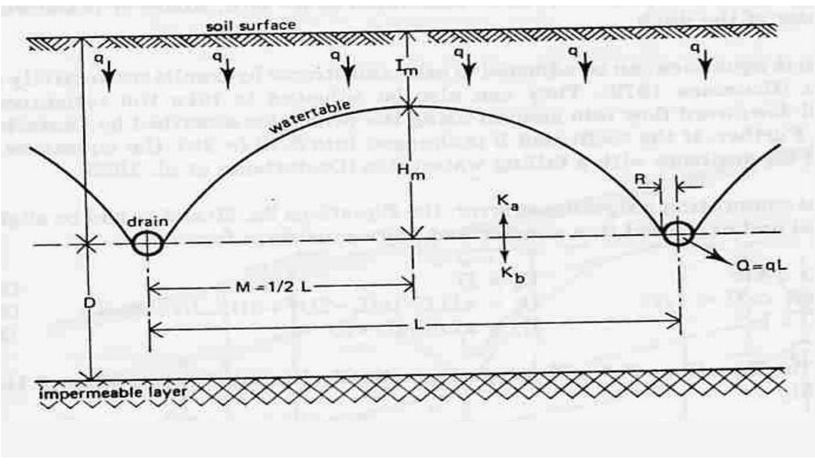

Figure 1. Illustration of the parameters involved in Hooghoudt's equation

3 DRAINAGE OF SLOPING LAND WITH ENTRANCE RESISTANCE

Figure 2 illustrates the parameters involved in the drainage of sloping land with entrance resistance. The symbols used in the figure are defined as follows:

- D is the depth of an impermeable layer below the drain centre (m);
- G is the depth of the drain centre below the soil surface (m);
- Ho is the entrance head, equal to the height of the water table just above the drain, measured from the centre of the drain (m);
- Hu is the height of the water table at the water divide between drains, measured in upslope direction from the centre of a drain (m);
- Hu'=Hu-Ho (Hu reduced with entrance head);
- Hd is the height of the water table at the water divide between drains, measured in down slope direction from the centre of the drain (m);
- Hd'=Hd-Ho (Hd reduced with entrance head);
- H* is the height of the water table midway between drains, measured from the centre of the nearest down slope drain (m);
- Hgr is the height of the water table midway between drains, measured from the sloping line through the centres of the drains (m);
- I is the depth of the water table below the soil surface midway between drains (m);
- L is the drain spacing (m);
- M=L/2 (half the drain spacing);
- S is the slope of the land (m/m);
- Zu is the distance to the water divide between drains, measured in upslope direction from a drain (m);
- Zd is the distance to the water divide between drains, measured in down slope direction from a drain (m).

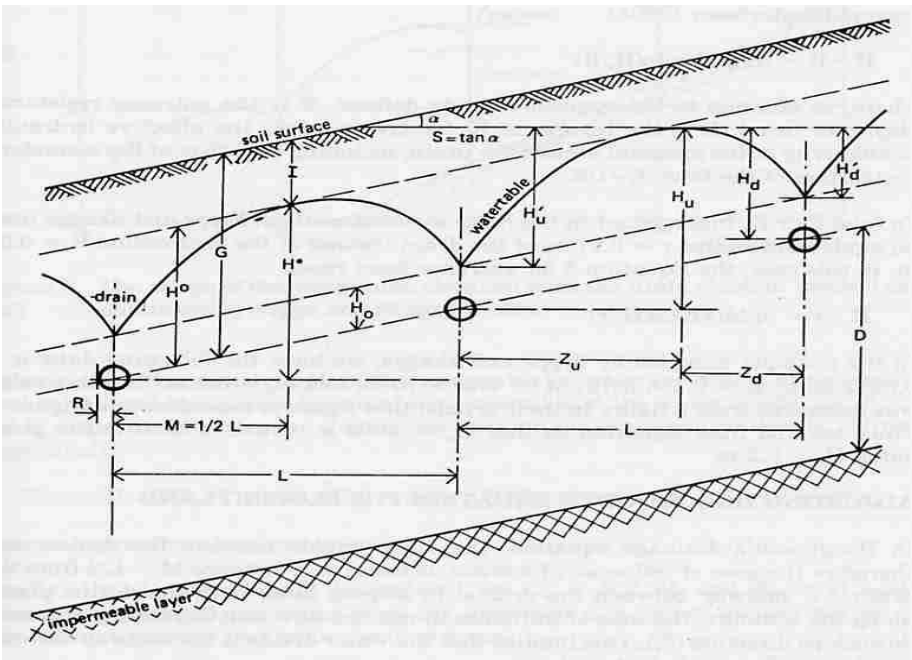

Figure 2. Illustration of the distance parameters involved in the drainage of sloping lands, with entrance resistance taken into account

4 ADJUSTING HOOGHOUDT'S EQUATION FOR ENTRANCE RESISTANCE

The equation of Hooghoudt can be adjusted to take into account the entrance resistance to the flow of groundwater around the drain, using the principles discussed by Oosterbaan et al. (1989, 1990a and b) and replacing in the equations:

- Hm by Hm'=Hm-Ho (Hm is reduced with entrance head)
- D by D" =D+Ho (D is increased with entrance head)
- R by R" =R+Ho (R is increased with entrance head)
- Ha by Ha'=(Hm-Ho)/2, where Ho is the entrance head as defined above.

The entrance head can be determined directly, or according to the radial flow concept (Oosterbaan 1990a):

$$Ho-R = (E.q.M/)\ln(Ho/R) \qquad (3)$$

where, in addition to the symbols already defined, E is the entrance resistance (day/m or hr/m). It is the inverse of Ke (m/day or m/hr), the effective hydraulic conductivity of the material around the drain, including the effect of the secondary contraction of the flow: E=1/Ke.

To take E or Ke into account in the finite element method, Fipps and Skaggs used an equivalent radius r=0.01m of the drain instead of the real radius R=0.05m. In this case, the Equation 3 for entrance head reads

$$Ho-r = (q.M/K)\ln(Ho/r) \qquad (3a)$$

In the example provided by Fipps and Skaggs, we have the following data: q=0.0022m/hr, K=0.158m/hr, M=L/2=15m, and Ho=0.2m. The last value was measured from a figure in their article that is reproduced in Figure 3. Thus we find from Equation 3a that Ho=0.209m, which is close to the given value Ho=0.2m.

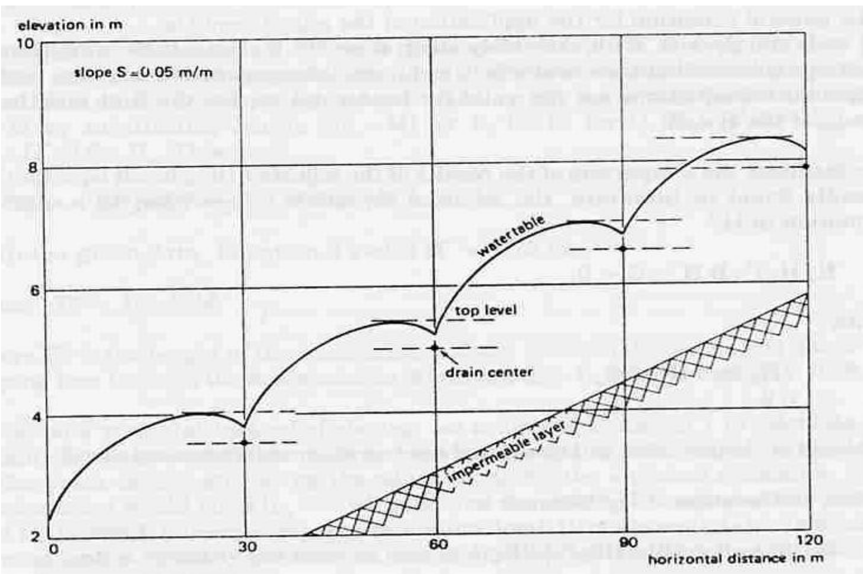

Figure 3. The shape of the water table obtained with the finite element method as published by Fipps and Skaggs (1989)

5 ADJUSTING HOOGHOUDT'S EQUATION FOR SLOPING LAND

In Hooghoudt's drainage equation, the water divide between the drains, and therefore the zone of influence of a drain, is found at a distance $M=L/2$ from the drain, i.e. midway between the drains. In sloping land, with the drains placed along the contours, the zone of influence in upslope direction (Zu) is larger than in down slope direction (Zd). This implies that the water divide is not midway between the drains (Figures 2, 4, 5, and 6).

However, Hooghoudt's equation (Equation 2) can be adjusted to sloping land by using the principles introduced by Oosterbaan (1975) by which L is either replaced by 2Zu or by 2Zd, so that Equation 1 is changed into a set of two equations.

The height (Hm) of the water table midway between the drains above drain level is then replaced by the height Hu of the water table at Zu ($Hu>Hm$) and by the height Hd at Zd ($Hd<Hm$) respectively.

Further, the average height Ha of the water table between the drains above drain level is replaced by half the height of the water table at the water divide (i.e. at Zu and Zd respectively) above the sloping line through the centres of the drains, giving respectively $Hau=(Hu-S.Zu)/2$ and $Had=(Hd+S.Zd)/2$, where S represents the slope (m/m).

Finally, the drain radius R is replaced by $Ru=R.Zu/M$ and $Rd=R.Zd/M$ respectively, because the water from the upstream side enters the drain over a proportionally larger part of its circumference. Now, the following relations exist

$$Zu+Zd = 2M \quad (4a)$$
$$Hu-Hd = S(Zu+Zd) \quad (4b)$$
$$Q = q(Zu+Zd) \quad (4c)$$

where Q is the steady discharge from the drain in m/day per m length of drain (m/day).

The general condition for the application of the adjustments is $Zu<L$ and $Zd>0$. With extremely steep slopes ($S>10\%$), these conditions are perhaps not met, but then drainage is not a realistic proposition anyway. Further, the adjusted equations are not valid for border drains, i.e. the first and the last drain of the system.

To facilitate the comparison of the results of the adjusted Hooghoudt equation with results found in literature, the adjusted Equation 1 is written as a quadratic equation in Hu':

$$Ka(Hu')^2 + B.Hu' - C = 0$$

with

$$B = 2Kb.Du'' - Ka.S.Zu$$
$$C = q.Zu$$

where Du'' is the value of De adjusted for entrance resistance and slope.
Thus the solution of Hu' becomes

$$Hu' = \{-B+(B+4Ka.C)\}/2Ka \quad (5)$$

6 COMPARISON OF RESULTS WITH THE FINITE ELEMENT METHOD

Figure 3 shows a reproduction of a figure presented by Fipps and Skaggs (1989). From the original figure, the following estimates can be made: Zu=24m, Zd=6m, Hu=1.8m, and Hd=0.3m. As the slope is S=0.05m/m, and the spacing is L=30m, Equations 4a and b can be verified. According to the data of Fipps and Skaggs, we have in addition: Ho=0.2m, D=2m, R=0.05m, so that the adjustments for entrance resistance consist of replacing:

Hu=1.8m by Hu'=Hu-Ho=1.6m
D =2m by D" =D+Ho =2.2m
R =0.05m by R" =R+Ho =0.25m

Also, since Zu=24m and M=L/2=15m, we replace R"=0.25m by Ru"=R".Zu/M=0.4 m

Further data given are q=0.0022m/hr, Ka=Kb=0.158m/hr.

Equation 5 now gives Hu'=1.68m, so that Hu=Hu'+Ho=1.88m. These values do not differ greatly from the given values Hu'=1.6m and Hu=1.8m respectively. The difference is 5%. It can therefore be concluded that the adjusted Hooghoudt equation describes the situation adequately.

The height of the watertable H* at a distance M=L/2, i.e. midway between the drains, and taken with respect to the level of the drain centre downslope, is to be found from an application of Equation 1 to the mainly horizontal flow in the region Zu-M substituting herein:

2(Zu-M) for L,
D+H* for D,
Hu-H* for Hm
(Hu-H*)/2 for Ha.

This gives:

$$H^* = \{(Hu+D)-q(Zu-M)/K)\}-D \qquad (6)$$

With the given data, Equation 6 yields H*=1.73m.
Using:

$$Hgr.=H^*-S.M \qquad (7)$$

where Hgr. is the height of the water table midway between the drains and above the sloping line through the drain centres (Figures 2 and 4), we find that Hgr.=0.98m.

If the land were flat instead of sloping, we could use Equation 1 to calculate the height Hm of the water table midway between the drains with respect to the level of the drain centre, still using the adjustments for the entrance resistance. Such a calculation would yield Hm=0.94m, so that the difference between Hgr.(=0.98m) and Hm is small. Consequently, the minimum depth (I) of the water table in drained sloping land is virtually the same as that in drained flat land under conditions that are otherwise the same.

Hence, the adjusted Hooghoudt equation gives results that are in line with the generally stated conclusion that there is no important error in water table depth when drainage systems in sloping land are designed as if the land were flat (Fipps and Skaggs 1989).

7. COMPARISON OF RESULTS WITH SAND TANK EXPERIMENTS

Figures 4 and 5 are reproductions of figures presented by Zeigler (1972). They are the outcome of experiments in a sand tank for a similar drainage situation as described earlier. Figure 4 shows the shapes of the water table in the tank with a slope S=0.025m/m at varying steady recharge rates ranging from q=0.00812 to 0.0161m/hr. It can be seen that the zone of influence Zu in upslope direction decreases as the recharge rate increases. Figure 5 depicts the shapes of the water table at varying slopes, ranging from S=0 to S=5%, when the steady recharge is q=0.0161m/hr. The figure shows that the zone Zu increases with increasing slope.

The following additional data were provided by Zeigler (1972): L=3.66m, D=0.61m, Ka=Kb=0.619m/hr. The drains were surrounded by a permeable envelope, making a total radius of R=0.061m. The entrance resistance cannot be clearly deduced from the figures. Yet, some entrance resistance is likely to have occurred, because the recharge rates are extremely high, ranging from q=195 to 386mm/day (!). It is therefore estimated that Ho=0.01m when q=0.00812m/hr and Ho=0.02m when q=0.0161m/hr. This gives an adjusted drain radius R"=R+Ho=0.07m and R"=0.08m respectively.

Now, with the adjusted Hooghoudt equation (Equation 5), the maximum height of the water table at the water divide can be calculated from the data. A selection of the cases reported by Zeigler is used. The comparison of the results of the calculation with the experimental results is given in Table 1.

Table 1 shows that the calculated Hu values are in good agreement with the experimentally measured values. The maximum difference (found in the column with S=0.05m/m and q=0.0161m/hr) amounts to 7%. It can therefore be concluded that the adjusted Hooghoudt equation also describes these situations adequately.

Table 1 also shows that the Hgr. values (i.e. the height of the water table midway between the drains above the line connecting the centre points of the drains, Figure (1), are virtually the same for all the cases with q=0.0161m/hr. This again confirms that the drainage of sloping land can be treated as if the land were flat

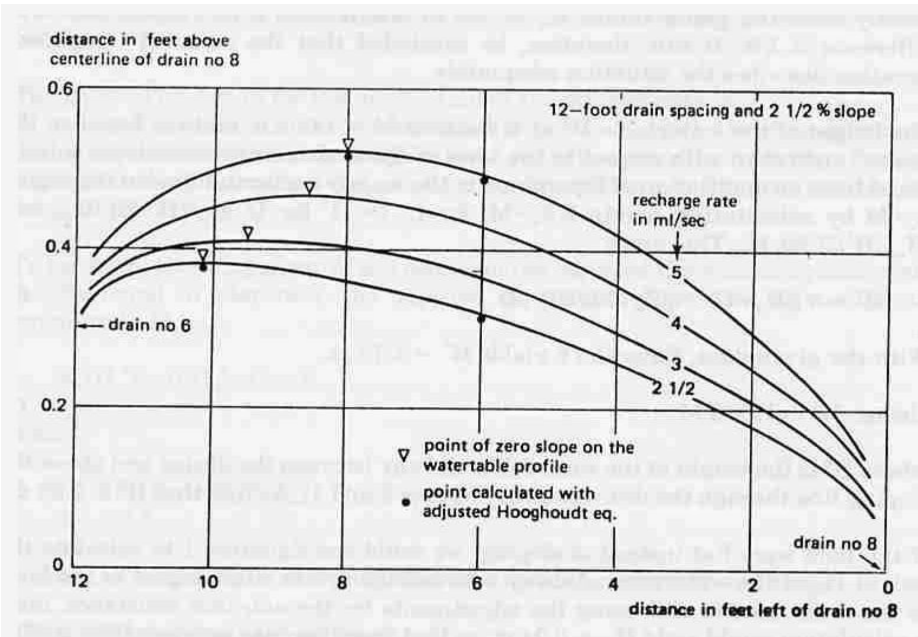

Figure 4. The height of the water table at varying recharge rates in sand tanks as reported by Zeigler (1972) and with calculated points

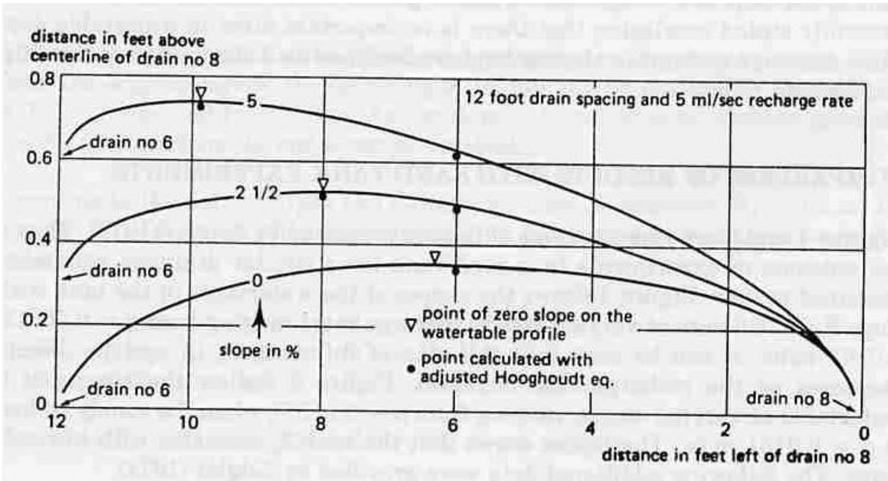

Figure 5. The height of the water table at varying slopes in sand tanks as reported by Zeigler (1972) and with calculated points

Table 1. Comparison of experimental results of drainage in a sloping sand tank (Zeigler 1972) with results of the adjusted Hooghoudt equation, using the same data

Parameter	Source	S=0.025 q=0.00812	S=0.025 q=0.0161	S=0.050 q=0.0161	S=0.000 (m/m) q=0.0161 (m/hr)
Zu, m	Fig 4&5	3.08	2.44	2.98	1.84
Hu, m	Fig 4&5	0.117	0.159	0.225	0.105
Hu, m	Eq 5	0.115	0.152	0.208	0.104
H*, m	Fig 4&5	0.100	0.151	0.194	0.105
H*, m	Eq 6	0.101	0.147	0.187	0.104
H°, m	Fig 4&5	0.054	0.105	0.102	0.105
H°, m	Eq 7	0.052	0.098	0.095	0.104

8. SUMMARY AND CONCLUSION

The drainage equation of Hooghoudt can be adapted to a wide variety of drainage conditions by small and simple adjustments. In this article, the equation was adjusted to take entrance resistance and land slope into account, yielding mathematical expressions that can be solved with a pocket calculator. With previously reported expressions, this was not possible. Until now, analysing the drainage of sloping land depended mainly on numerical or scale models. The adjusted equations have been tested with data from literature on sand tank and finite element models, under identical drainage conditions. The differences were small, which indicates that the adjusted Hooghoudt is quite accurate.

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