Free EnDrain software designed to calculate parameters of agricultural subsurface drainage systems using the energy balance of groundwater flow

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Abstract. For subsurface drainage systems by drain pipes or ditches in agricultural lands one may whish to calculate the drain spacing, discharge, level of the water table or the hydraulic conductivity of the soil. The free EnDrain software can perform these tasks. The common drainage equations use the height of the water table above drain level midway between the drains. EnDrain, however uses a method to find the entire shape of the water table from the drains themselves the midpoint between the drains. This requires an elaborate simulation technique different from the straightforward procedure used in the common equations, making the digital automation desirable. Furthermore it provides the options to perform the computation either based on the standard Darcy equation of groundwater flow or on the modern energy balance of groundwater flow. In the latter case the solutions have to be made numerically with numerous iterations so that the use of a computer model is unavoidable. In this article the use of Endrain will be explained and examples will be given using data from literature. The results will be compared with the results in literature, which implies that only the level of the water table at the midpoint between the drains can be checked, and when necessary the differences will be explained.

Contents

1. Introduction

The application of equations for subsurface drainage systems by drain pipes and ditches is widespread. Amongst these, the equation of Hooghoudt is well known [*Reference 1*]. The standard equations are based on the Darcy equation for groundwater flow and deal with the level of the water table midway between the drains only.

To enhance more elaborate solutions producing the entire shape of the water table from the drains themselves the midpoint between the drains, as well as to apply the complete energy balance of groundwater flow [*Reference 2, Reference 3*] instead of the more elementary Darcy equation, the free EnDrain [*Reference 4*] software has been developed.

For that reason, and for demonstrating the differences between the customary methods of computation and those of EnDrain, this introductory section is divided into two parts: 1.A for the traditional procedures and 1.B for the EnDrain principles.

1.A The common procedure

In practice the Hooghoudt equation reads:

$$Q = [\ 8\ Kb.De.Hn + 4\ Ka.Hn^2\ ]\ /\ S^2 \qquad \text{[Eq. 1]}$$

where Q = drain discharge (m/day), Kb = hydraulic conductivity below drain level (m/day), Ka = hydraulic conductivity above drain level (m/day), De = the equivalent depth of the impermeable layer below drain level (m), Hn = net height of the water table above drain level midway between the drains (m), and S = drain spacing (m). The net height equals Hm – He with Hm =real height of the water table above drain level midway between the drains (m), and He the height at the drains, representing entrance resistance.

The actual depth (Da) of the impermeable layer below drain level is demonstrated in *figure 1* equaling D2 – Db. The value of the equivalent depth De depends on Da, the width of the drain (W) and the drain discharge Q.

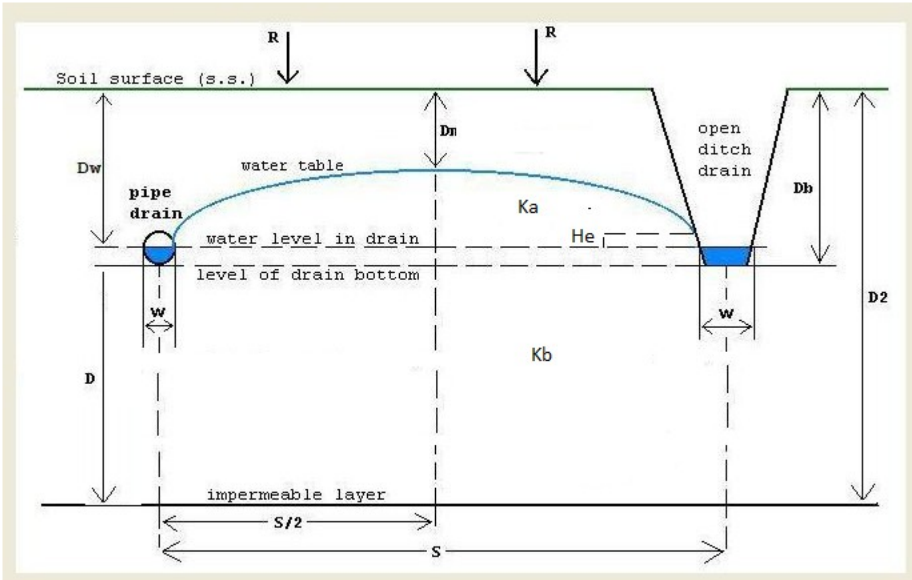


Figure 1. *Screen print of an illustration of parameters in a subsurface drainage system with pipe drains or ditches*.

The equivalent depth De can be computed from (*Reference 5*) :

$$De = \pi S / 8 \; \{ \ln (S / ½\pi W) + F(x) \} \qquad [Eq. 2]$$

where:

$X = 2\pi \, Da$

$F(x) = \Sigma \, (4e^{-2nX}) \; / \, n \, (1 - e^{-2nX})$

with: n = 1, 3, 5, . . . and e = 2.71 . . , the basis of the Niperian logarithm.

The computation of the converging series F(x) is cumbersome and computer program like EnDrain facilitates the operation. ‘

Knowing Q, Kb, De, Ka and S makes it possible to calculate hydraulic head Hn. When Q, Kb, De,, Ka and Hm are given, one can compute the spacing S. With known values of Kd, De Hn and S the discharge Q van be found. Given a series of measured data on Q and Hn, one can find Kb, Ka and De (*Reference 6. Reference 7, Appendix I*).

1.B The simulative procedure

The flow of groundwater towards a subsurface drain above a deep impermeable layer consists of mainly horizontal flow from the midpoint between the drains, but at a certain distance from the drain the groundwater flow converges toward the drain. This convergence is called radial flow [*Reference 1, Figure2*]. In the area of radial flow, the cross-section of the flow at a distance X from the drains is formed by the circumference of a quarter circle with a length Y=½πX. This principle is conceptualized in Figure 2 by letting an imaginary impermeable layer slope away from the centre of the drain at an angle with a tangent ½π.

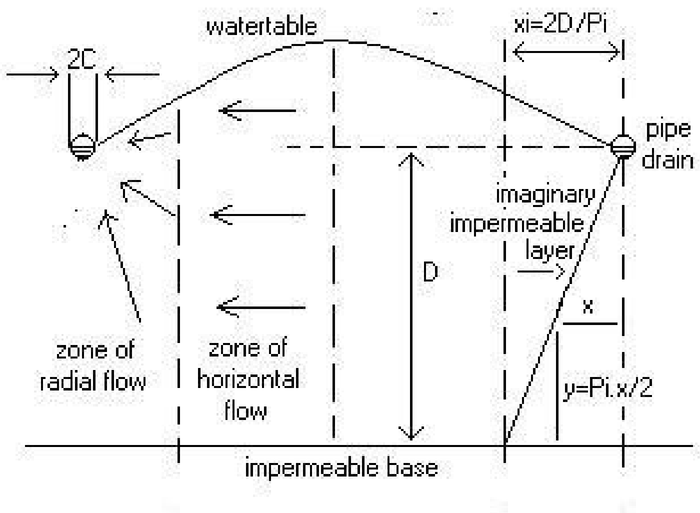


Figure 2. *The division of groundwater flow towards a subsurface drain into a part with horizontal flow and a part with radial flow (left side of the figure). The radial flow is simulated by an imaginary impermeable layer starting at a distance W=Xi=2D/π until reaching the drain itself (right side of the figure). Here D is the depth of the impermeable layer below drain level.*

Using the energy balance of groundwater flow [*Reference 2, Reference 3 and Appendix II *)*], the height of the water table above drain level (Hx) can be found from the rainfall (recharge, R), N=0.5 S (S being the drain spacing), K (the hydraulic conductivity) , the distance from the drain (X), the width of the zone of radial flow (W=Xi), the height of the water table above drain level in the midpoint between the drains Hn, and the radius of the drain (C) as:

$C<X<W$:

$$Hx = \int_C^X \left[\frac{R(N-X)}{K(Hx+\frac{1}{2}\pi X)}\right] dX - \int_C^X \left[\frac{Hn-Hx}{N-X}\right] dX$$

$W<X<N$:

$$Hx = \int_C^X \left[\frac{R(N-X)}{K(Hx+D)}\right] dX - \int_C^X \left[\frac{Hn-Hx}{N-X}\right] dX$$

The second term in the above equations represents the energy associated with the incoming recharge.

In addition, EnDrain accounts for entrance resistance, layered anisotropic soils, each layer having a vertical hydraulic conductivity that may differ from the horizontal one [*Reference 3*].

For the calculation of the hydraulic head Hx a numerical iterative computation method is needed because the height Hn is not known beforehand.

*) *Appendix II describes the principles of the energy balance of groundwater flowing and gives the derivation of the equations on this page*

2. The use of EnDrain

Figure 3 shows the input menu (user interface) of EnDrain (*Reference 4*).

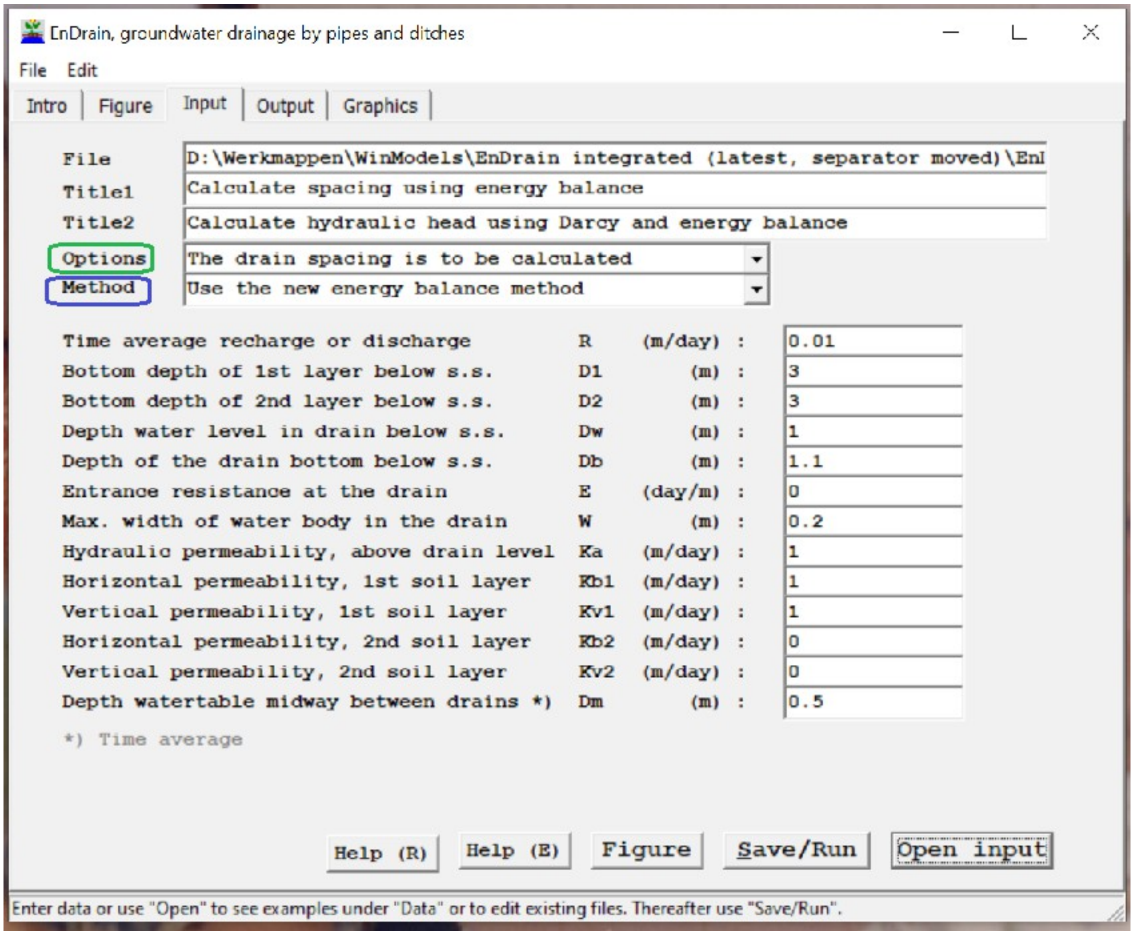


Figure 3. *Input menu of EnDrain. The purpose of the computation in this case is to calculate the drain spacing, see the option (green square) which is the calculation of the drain spacing, while the method selected is the modern energy balance of groundwater flow (blue square). The required data are depicted in the table.*

Figure 4 illustrates the calculation options available.

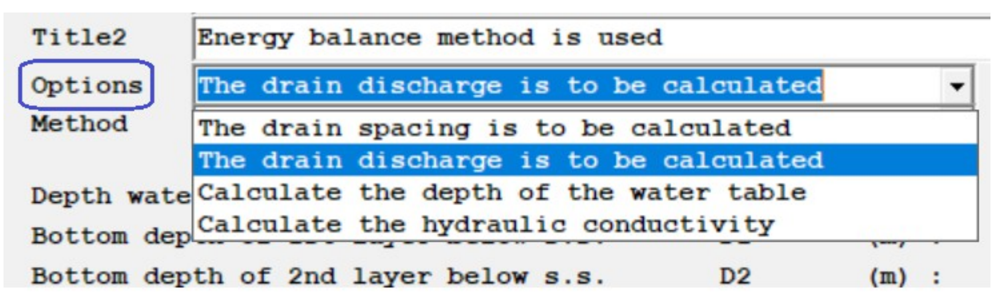


Figure 4. *Showing the available calculation options where the calculation of the discharge is chosen.*

Figure 5 depicts the two methods possible: new and classical.

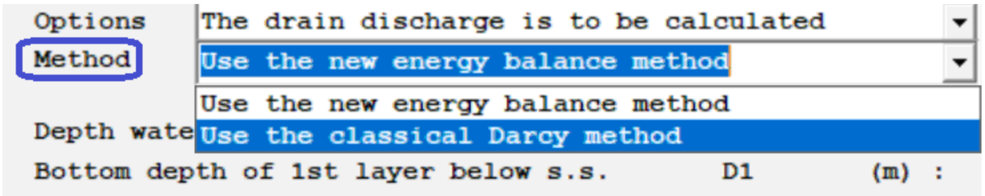


Figure 5. *The present method of the new energy balance is to be changed into the classical Darcy method.*

Figure 6 clarifies the different results between the new energy balance method and the classical Darcy method. The modern energy balance method yields a deeper water table compared to the classical Darcy method. The reason is that the new method takes into account the energy associated with the incoming percolation water (R in figure 1) so that more energy is available to overcome the resistance of the saturated ground water flow to the drains and the height of the water table above drain level can be less (*Reference 6, Reference 7*).

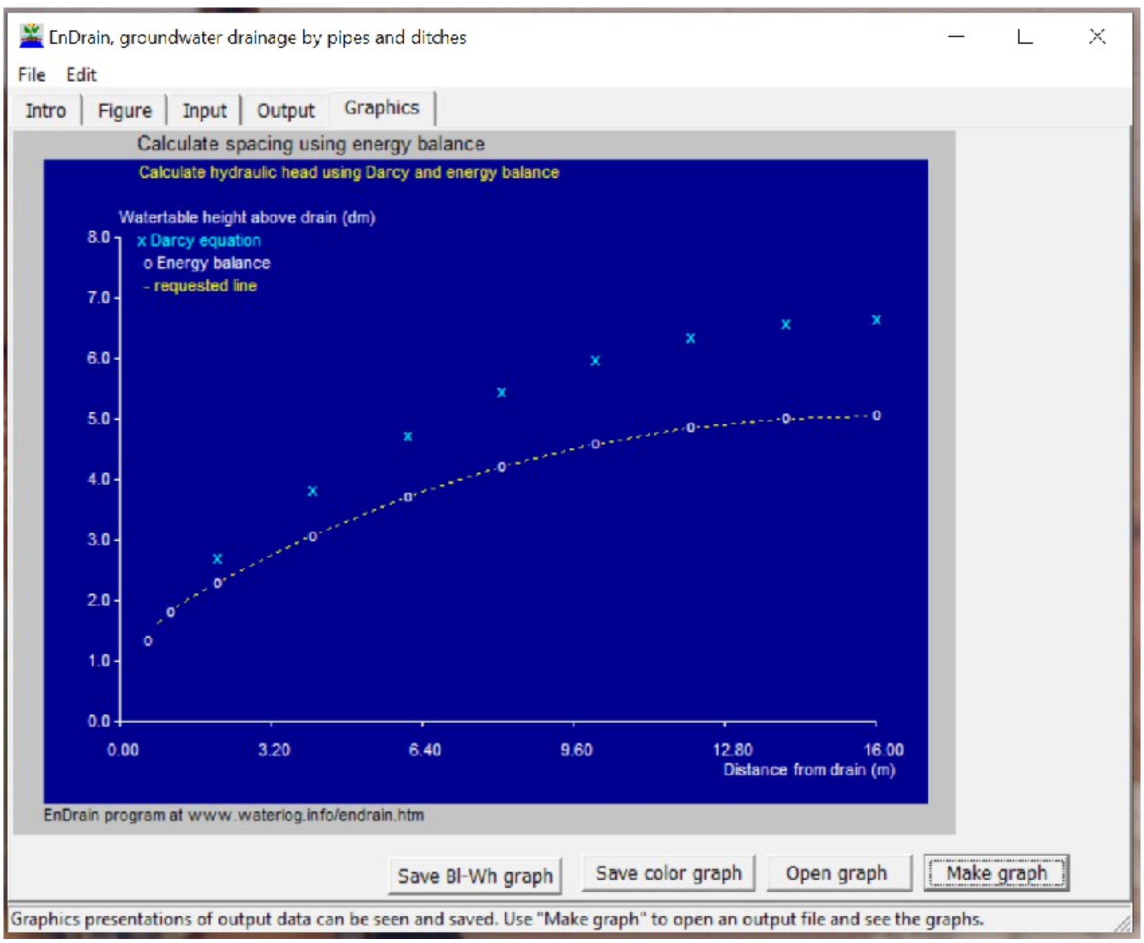


Figure 7. *The results for computation of the water table height is different for the two methods. The blue crosses represent the classical Darcy method while the yellow dotted curve signifies the modern energy balance method.*

3. Four Examples

Example 1.

Ritzema [*Reference 1*] provided the following data:

> Discharge Q = 1 mm/day = 0.001 m/day, height of the water table above drain level Hn =1.0 m, drain radius C = 0.1 m, W = 2C = 0.2 m, hydraulic conductivity K=Ka=Kb=0.14 m/day and depth of the impermeable layer below drain level D = 4.8 m.

With these data he found by trial and error, assuming different initial drain spacings L, that the equivalent depth De is 3.22 m and the drain spacing S = 65 m.

EnDrain calculates a spacing of 67 m using the current Darcy method The result of Ritzema is close enough. Further, EnDrain shows the following graph for the shape of the water table from the drain to the midpoint between the drains

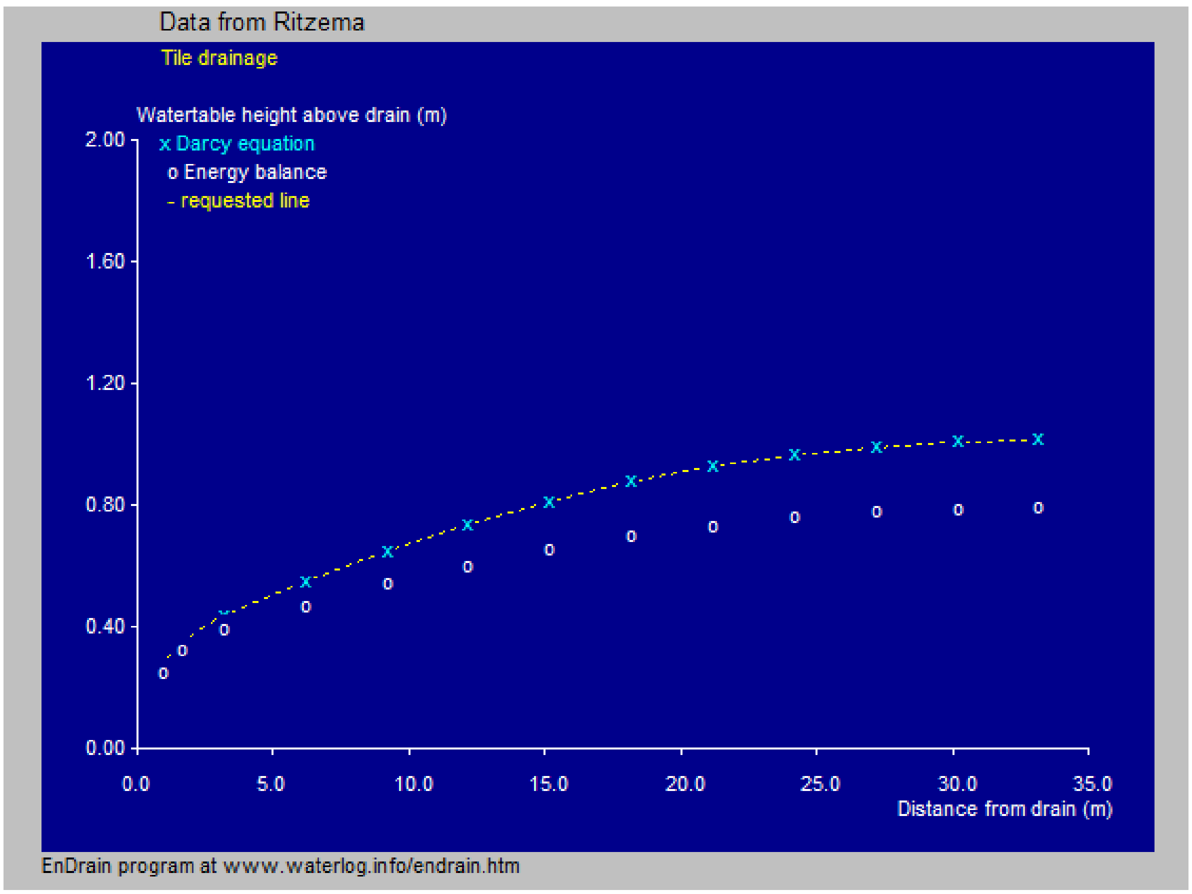
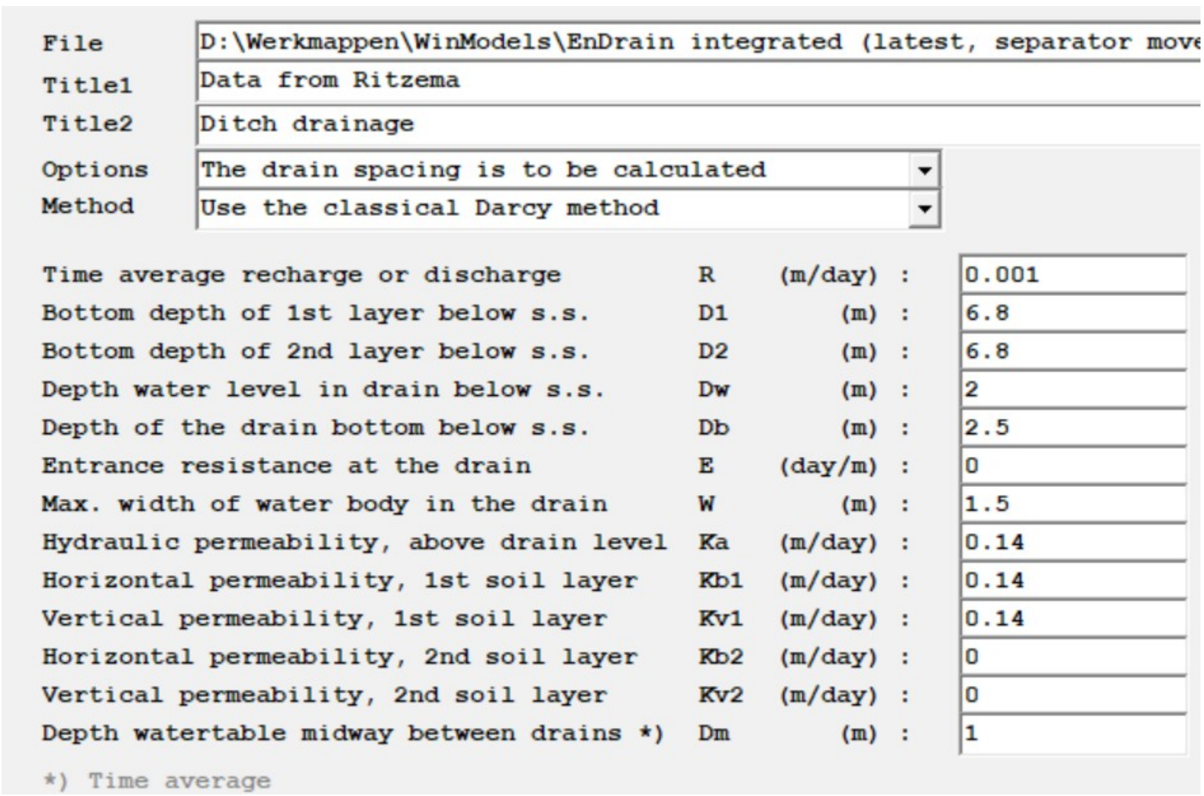


Figure 7. *The results for computation of the water table height in the case of Ritzema's example 1. The blue crosses represent the classical Darcy method used while the yellow dotted curve signifies the modern energy balance method. The water level in the second case is deeper than in the first, owing to the gain in hydraulic energy from the downward recharge If the spacing would have been calculated with the energy method, the drain spacing would have been larger, reducing the installation costs*

Example 2

Example 2 uses the same data as example 1, with the only difference that ditches are used instead of tile drains reaching a depth of 2 m having a bottom width of 0.5 m, a side slope of 1:1, while the water level in the drain is 0.5 m above the bottom. For these condition he calculates the wetted perimeter of the ditch to be 1.91 m for use as ½πW in the equation for equivalent depth [Eq. 1]. After a trial and error procedure, he determined the required ditch spacing to be 72 m, a liitle more than that for tile drains (65 m).

The input menu of EnDrain for this situation looks like in *figure 8* below.



File	D:\Werkmappen\WinModels\EnDrain integrated (latest, separator move
Title1	Data from Ritzema
Title2	Ditch drainage
Options	The drain spacing is to be calculated
Method	Use the classical Darcy method

Time average recharge or discharge	R	(m/day) :	0.001
Bottom depth of 1st layer below s.s.	D1	(m) :	6.8
Bottom depth of 2nd layer below s.s.	D2	(m) :	6.8
Depth water level in drain below s.s.	Dw	(m) :	2
Depth of the drain bottom below s.s.	Db	(m) :	2.5
Entrance resistance at the drain	E	(day/m) :	0
Max. width of water body in the drain	W	(m) :	1.5
Hydraulic permeability, above drain level	Ka	(m/day) :	0.14
Horizontal permeability, 1st soil layer	Kb1	(m/day) :	0.14
Vertical permeability, 1st soil layer	Kv1	(m/day) :	0.14
Horizontal permeability, 2nd soil layer	Kb2	(m/day) :	0
Vertical permeability, 2nd soil layer	Kv2	(m/day) :	0
Depth watertable midway between drains *)	Dm	(m) :	1

*) Time average

Figure 8. *Part of the input menu of EnDrain with definition of input data for the case of Ritzema's example 2. The width of the water surface in the ditch is 1.5 m.*

For this case EnDrain reveals a spacing of 77 m as can be seen in the lower pare of the output file (*Figure 9*). The result of Ritzema (72 m) is close enough.

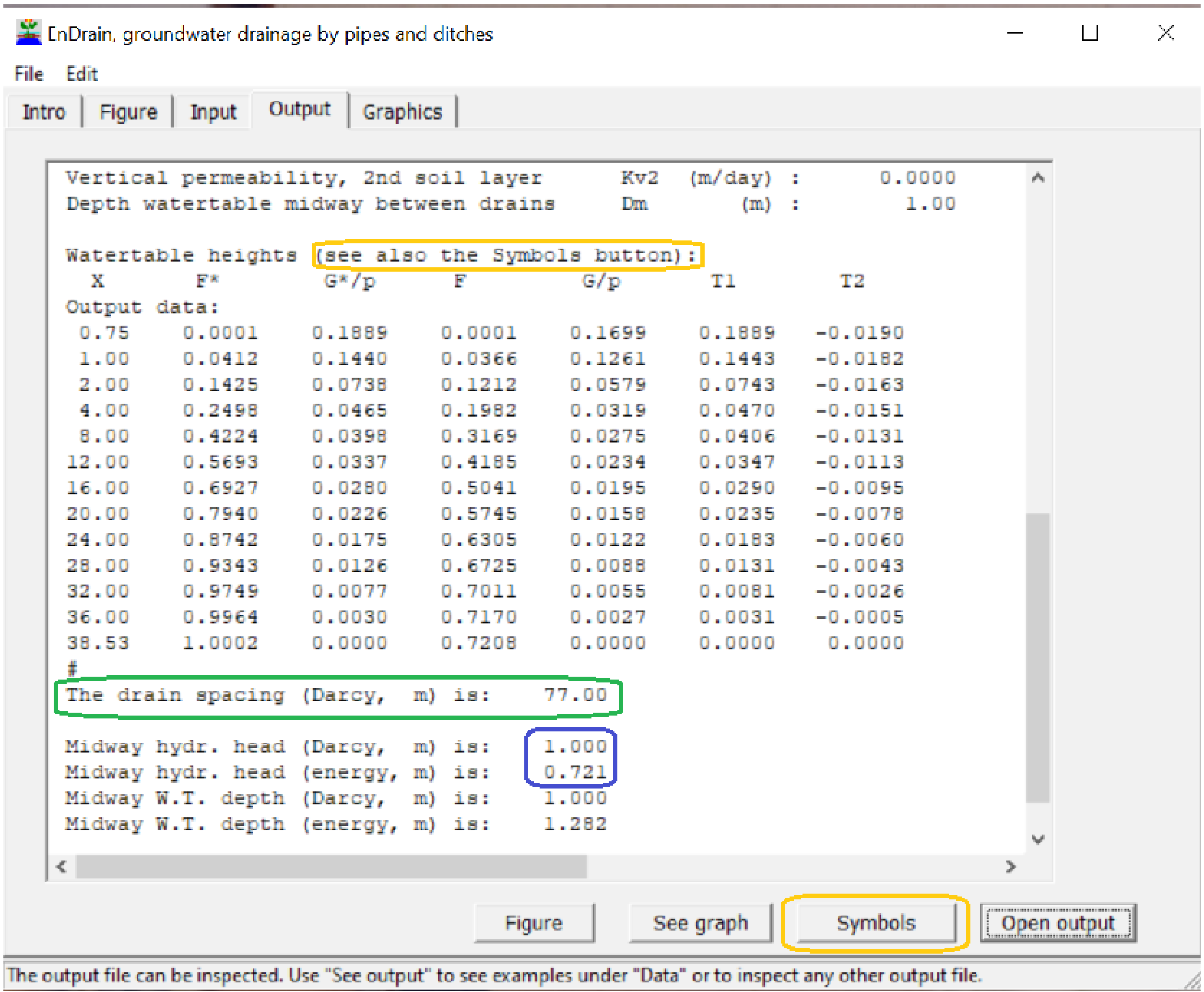


Figure 9. *Lower part of the output menu for the case of Ritzema's example 2 referring to ditch drains. The computed drain spacing is 77 m (green square). The height of the water table(hydraulic head) above the drain level midway between the drains for the standard Darcy method is 1 m (blue square) as should be according to the input, while the energy balance method yields a lower head (0.721 m, blue square), meaning that the spacing could be wider when the head is taken as 1 m used for the Darcy method. The symbols used in EnDrain are different from those in this paper, but they are explained as indicated by the orange squares.*

<u>Example 3</u>

The single soil layer in *example 1* is replaced by two soil layers with interface exactly at drain depth. The hydraulic conductivity Ka op the top layer is only 0.06 m/day and that of the lower layer is Kb = 0.30 m/day, which is higher than the 0.14 m/day in *example 1*.

Using a number of trails and errors on the basis of a variety of tentative L values, Ritzema finally found that the required spacing is 95 m. This distance between the drains is longer than that in the previous examples because the Kb value has been increased.

The Endrain results, depicted in the following table, mention a required spacing of 98 m The result of Ritzema (95 m) is close enough.

Table 1. *Screen print of part of the EnDrain output menu, manifesting a drain spacing of 98 m. The height of the water table(hydraulic head) above the drain level midway between the drains for the standard Darcy method is 1 m as should be according to the input, while the energy balance method yields a lower head (0.736 m), meaning that the spacing could be wider when the head is taken as 1 m used for the Darcy method.*

```
#
The drain spacing (Darcy,  m) is:     98.06

Midway hydr. head (Darcy,  m) is:     1.000
Midway hydr. head (energy, m) is:     0.736
Midway W.T. depth (Darcy,  m) is:     1.000
Midway W.T. depth (energy, m) is:     1.267
#
```

Example 4.

In this example given by Ritzema, the drain is situated entirely inside a first layer of with a relatively low hydraulic conductivity underlain by a second layer with a higher conductivity. In that case, for the calculation by hand, the Hooghoudt equation is not applicable, therefore the Ernst equation [*Reference 8*] was used. It reads:

$$Hn = Q \, [\, (D_1 - Dw)/Ka + L^2 / 8 \, \{Kb \, (D_2 - D_1) + \{L / \pi \, Ka\} \ln \{ \, (D_1 - Dw) \; / \tfrac{1}{2} \pi \, W \, \} \,]$$

The data employed by Ritzema are:

Discharge Q = 7 mm/day = 0.007 m/day, height of the water table above drain level Hn =0.7 m, drain radius C = 0.05 m, W = 2C = 0.1 m, hydraulic conductivity of the first layer Ka=0.5 m/day, hydraulic conductivity of the second soil layer Kb=2.0 m/day, depth below soil surface of the water level in the drain Dw = 1.0 m, depth below soil surface of the first soil layer D_1 = 2.0 m, depth below soil surface of the second layer D_2 = 6.0 m. At this depth the impermeable layer is found.

Entering these data into the Ernst equation yields a quadratic equation in L:

$$0.014 \, L^2 + 1.18 \, L - 98.6 = 0$$

From which follows L = 51.8 m.

For the Darcy case, the EnDrain program yields L = 50.5, so that the result of the Ernst equation is not much different. On the other hand, the energy balance equation available in EnDrain produces L = 56.9 m. This spacing is wider and cheaper thanks to incorporating the energy supplied by the downward percolation water upon entering the water table.

3. Conclusions

The standard procedures for the calculation of the drain spacing for the design of agricultural subsurface drainage project are cumbersome. The EnDrain software facilitates the procedure to a great extent. Comparison of EnDrain outcomes in three documented cases with those that were laboriously calculated by hand using the current equations based on Darcy's law, reveals close correspondence.

The common procedure only handles the level or depth of the water table midway between the drains, while EnDain can simulates the entire curve of the water table from drain to the midpoint between the drains.

In addition, EnDrain can make use of the energy balance of groundwater flow instead of the straightforward formulas based only on the Darcy principle, which leads to larger, cost saving, spacings .

4. References

Reference 1.
H.P. Ritzema, 1994. *Subsurface Flow to drains*. Chapter 8 in: H.P. Ritzema (editor), Drainage Principles and Applications, ILRI Publication 16. International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands. On line:
http://www2.alterra.wur.nl/Internet/webdocs/ilri-publicaties/publicaties/Pub162/pub162-h1.0.pdf
or:
https://www.researchgate.net/publication/272483377_Subsurface_flow_to_drains

Reference 2.
R.J. Oosterbaan, J. Boonstra and K.V.G.K. Rao. *The energy balance of groundwater flow*. Paper published in V.P.Singh and B.Kumar (eds.), 1996, Subsurface-Water Hydrology, p. 153-160, Vol.2 of Proceedings of the International Conference on Hydrology and Water Resources, New Delhi, India, 1993.Kluwer Academic Publishers, Dordrecht, The Netherlands.
On line: https://www.waterlog.info/pdf/enerbal.pdf
Or:
https://www.researchgate.net/publication/332470225_The_energy_balance_of_groundwater_flow

Reference 3.
The energy balance of groundwater flow *applied to subsurface drainage in anisotropic soils by pipes or ditches with entrance resistance.* Paper based on: R.J. Oosterbaan, J. Boonstra and K.V.G.K. Rao, 1996, "The energy balance of groundwater flow". Published in V.P.Singh and B.Kumar (eds.), Subsurface-Water Hydrology, p. 153-160, Vol.2 of Proceedings of the International Conference on Hydrology and Water Resources, New Delhi, India, 1993. Kluwer Academic Publishers, Dordrecht, The Netherlands. ISBN: 978-0-7923-3651-8
On line: https://www.waterlog.info/pdf/enerart.pdf
or:
https://www.researchgate.net/publication/332470086_THE_ENERGY_BALANCE_OF_GROUNDWATER_FLOW_APPLIED_TO_DITCH_DRAINAGE_IN_ANISOTROPIC_SOILS

Reference 4.
EnDrain, free software for drainage system calculations using four options and two methods.
On line: https://www.waterlog.info/endrain.htm

Reference 5.
Van Der Molen, W.A. , and J. Wesseling, 1981. *A solution in closed form and a series solution in Hooghoudt's drain spacing formula*. In: Agricultural Water Management 52, pp 336 - 346.

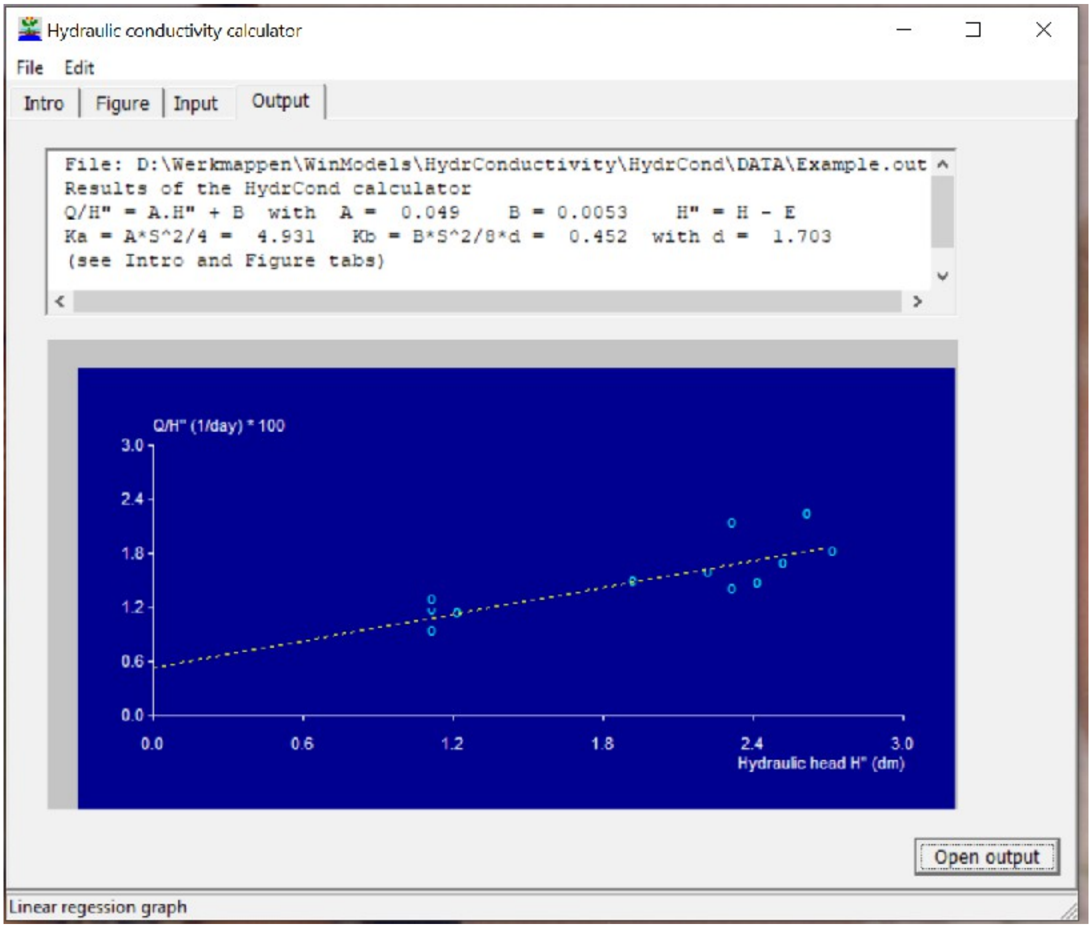
Reference 6.
HydrCond, free software for the computation of Ka, Kb and De given a series of measured data on Q and Hn. On line: https://www.waterlog.info/zip/HydrCond.zip .
An example is shown in the *Appendix* I

Reference 7.
Determination of the soil's hydraulic conductivity based on measurements of drain discharge and level of the groundwater table in agricultural subsurface drainage system using free software.
On line: https://www.waterlog.info/pdf/HydrCond.pdf

Reference 8.
L.F. Ernst, 1956. Calculation of the steady flow of groundwater in vertical cross-sections. Netherlands Journal of Agricultural Science, pp. 126-131. On line:
https://library.wur.nl/ojs/index.php/njas/article/view/17793

Appendix 1. Example of the results of the free HydrCond program

The figure below shows the results of the HydrCond program given levels of the water table and drain discharge in a subsurface drainage system in agricultural land.



Screen print of the output of the HydrCond program computing hydraulic conductivity Ka and Kb as well as the equivalent depth of the impermeable layer De, given a series of measurements of hydraulic head (height of the water table above drain level) and drain discharge.

Appendix 2. Principles of the energy balance of groundwater flow [*Reference 2*] and derivation of the equations used in section 1.B: *"The simulative procedure"*

1. ENERGY BALANCES

1.1 Energy fluxes

The hydraulic potential (P) can be defined as the energy per unit volume of water (ε / m^3, where ε represents energy units). The flow velocity (V) of groundwater can be defined as the discharge per unit cross-sectional area perpendicular to the direction of flow (m^3 / day per m^2). The product P.V therefore represents an energy flux, i.e. an energy flow per unit cross-sectional area (ε/day per m^2).

Figure A shows a longitudinal section of two-dimensional groundwater flow (i.e. the flow pattern repeats itself in the planes parallel to the plane of the drawing) in a phreatic aquifer, i.e. an aquifer with a free water table. The water table is recharged by downward percolating water (R m/day) stemming from rainfall and/or irrigation. A coordinate system, with X (m) giving the horizontal and Z (m) the vertical distance from the origin, is also indicated. The horizontal component of the flow velocity in any point (X,Z) is indicated by Vx. The Z-levels of the impermeable layer and the water table in a vertical cross-section are shown as I and J respectively.

The total energy flow through a vertical cross-section (Ex, ε/day per m width in the direction perpendicular to the longitudinal section) is

$$Ex = \int_I^J [\ Vx\ (P-Pr)\ ]\ dZ \qquad (1)$$

where Pr is a reference value of P, independent of X and Z, to be determined in accordance to the boundary conditions of the flow.

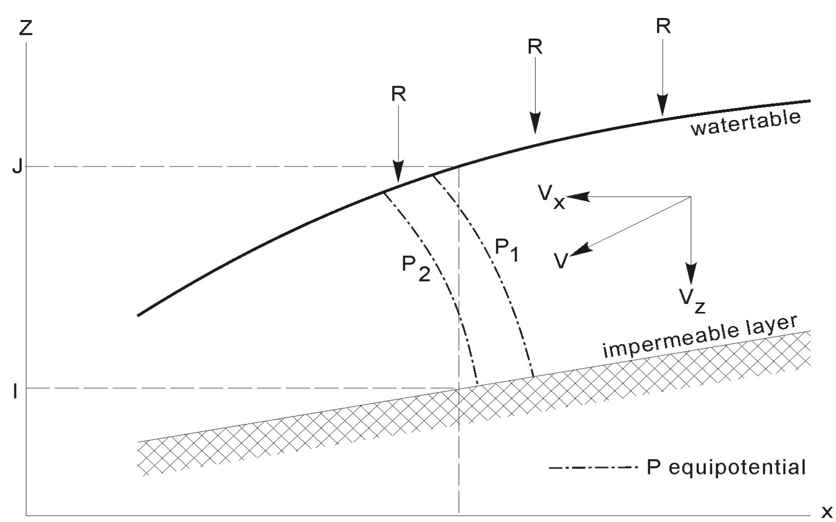


***Figure A**. A vertical cross-section in a longitudinal section along two-dimensional groundwater flow in a phreatic aquifer recharged by percolation.*

The change of the energy flow Ex per unit distance in a horizontal direction is

$$\frac{d\,Ex}{dX} = \frac{d}{dX} \int_{I}^{J} [\ Vx\ (P\text{-}Pr)\ ]\ dZ \qquad (1)$$

Using Leibnitz's rule, and assuming that the impermeable layer is horizontal (I=0, dI/dX=0),the above equation can be written as

$$\frac{d\,Ex}{dX} = \int_{0}^{J} [\frac{d}{dX} \{\ Vx\ (P\text{-}Pr)\ \}]\ dZ + Vj\ \ (Pj\text{-}Pr) \frac{dJ}{dX}$$

where Vj and Pj are the values of Vx and P at the water table.

Partial differentiation of the product Vx (P-Pr) in the previous equation, and noting that dPr / dX=0, yields

$$\frac{d\,Ex}{dX} = \int_{0}^{J} (Vx \frac{dP}{dX})\ dZ + \int_{0}^{J} [\ (P\text{–}Pr) \frac{dVx}{dX}\ ]\ dZ + Vj\ (Pj\text{–}Pr) \frac{dJ}{dX} \qquad (2)$$

1.2. The hydraulic head

The energy units **ε**, expressed in S.I. units, are $kg.m^2/day^2$, so that the units **ε** / m^3 of the potential P become kg / day^2 per m. The potential P can be converted into an hydraulic head as follows

$$H = P\ /\ \rho.g$$

where:

H is the hydraulic head (m)
ρ is the mass density of water (kg / m^3)
g is the gravitational acceleration (m / day^2)

With the conversion of potential P into head H, equation 2 becomes:

$$\frac{d\,Ex/dX}{\rho.g} = \int_{0}^{J} (Vx \frac{dH}{dX})\ dZ + \int_{0}^{J} [\ (H\text{–}Hr) \frac{dVx}{dX}\ ]\ dZ + Vj\ (Hj\text{–}Hr) \frac{dJ}{dX} \qquad (3)$$

From elementary hydraulics we know that the head H consists of three components: the elevation head (H_Z=Z), the pressure head (H_P) and the velocity head (H_V). The velocity head of groundwater flow is negligibly small, so that H=Z+H_P. At a phreatic surface, i.e. at the free water table, the pressure head corresponds to atmospheric pressure, which can be taken as zero reference pressure, so that H_P=0. Hence, for Z=J, i.e. at the water table, we find $H_{Z=J}$=J.

Using the Dupuit assumptions that the velocity Vx, the head H, and the gradient dVx / dX are constant with height Z, so that Vj=Vx and $H=H_{Z=J}$ = J, and writing Jr for Hr, Equation 3 can be simplified to

$$\frac{dEx/dX}{\rho.g} = (Vx.J)\frac{dJ}{dX} + J\,(J\text{–}Jr)\frac{dVx}{dX} + Vx\,(J\text{–}Jr)\frac{dJ}{dX} \qquad (4)$$

The assumption that the horizontal velocity Vx is constant with height is realistic when the resistance to vertical flow is small compared to that to horizontal flow.

1.3. The water balance

When the velocity Vx is constant with height Z, the two-dimensional discharge Q (m^3/day per m width of cross-section, or m^2/day) equals Q=Vx.J, and its differential coefficient, i.e. the change of discharge Q per unit change in distance X, becomes:

$$\frac{dQ}{dX} = \frac{d\,(Vx.J)}{dX} = Vx\frac{dJ}{X} + J\frac{dVx}{dX} = R$$

Hence, Equation 4 can be simplified to:

$$\frac{d\,(Ex\,/\,dX\,)}{\rho.g} = (Vx.J)\frac{dJ}{dX} + R\,(J\text{–}Jr) \qquad (5)$$

1.4. Energy conversion by friction of flow

The electric current in a conduit is known to lose electrical energy by its conversion to heat. The conversion rate is proportional to the resistance of the conduit and the square value of the current (the law of Joule). The resistance is inversely proportional to the conductance. In analogy, the conversion of hydraulic energy to friction of flow (Fx) is taken as

$$Fx = \int_0^J \left[\frac{(Vx)^2}{Kx}\right] dZ$$

Where Kx is the horizontal hydraulic conductivity of the soil (m/day). Further, it is stated that the energy loss rate (as in Equation 5) is proportional to the negative value of the friction losses. Thus we obtain:

dE / dX

$$\frac{\qquad}{\rho . g} = - Fx$$

Combining the previous two equations, and assuming again that the velocity Vx is constant with height Z, one obtains

$$\frac{d\,Ex / d\,X}{\rho . g} = - J \frac{(Vx)^2}{Kx} \qquad (6)$$

1.5. The hydraulic energy balance for steady state flow

When there is no change in storage of water, and consequently there is no change in storage of hydraulic energy (i.e. energy storage associated with water storage), we have a steady state: the hydraulic energy losses are fully converted into frictional energy. It can then be found from Equation 5 and 6 that:

$$Vx \frac{dJ}{dX} + \frac{R(J-Jr)}{J} = - \frac{(Vx)^2}{Kx}$$

The minus sign in the above equation assures that the energy losses are positive when the gradient dJ/dX is negative, which occurs when the flow Vx is positive (i.e. in the positive x-direction or, in *Figure A*, to the right), and vice versa. Division by Vx and rearrangement gives:

$$\frac{dJ}{dX} = - \frac{Vx}{Kx} - \frac{R(J-Jr)}{Vx.J} \qquad (7)$$

At the distance $X \leq N$, the discharge of the aquifer equals $Q = -R(N-X)$ (m^2/day) where the minus sign indicates that the flow is contrary to the X direction. From this water balance we find
$Vx = Q/J = -R(N-X) / J$ (m/day). With this expression for the velocity Vx, Equation 7 can be changed into:

$$\frac{dJ}{dX} = \frac{R(N-X)}{Kx.J} - \frac{Jr-J}{N-X} \qquad (8)$$

Introducing the drain radius C (m), and integrating Equation 8 from X=C to any value X, gives:

$$Hx = \int^{X} \left[\frac{R(N-X)}{\qquad} \right] dX - \int^{X} \left[\frac{Jr-J}{\quad} \right] dX \qquad (10)$$

C Kx.J C N

Figure B shows the vertically two-dimensional flow of ground water to parallel pipe drains with a radius C (m), placed at equal depth in a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation (R>0, m/day). The impermeable base is taken horizontal with a depth D>C (m) below the centre point of the drains. At the distance X=N (m), i.e. midway between the drains, there is a water divide. Here the water table is horizontal.

We consider only the radial flow approaching the drain at one side, because the flow at the other side is symmetrical, and also only the flow approaching the drain from below drain level.

According to the principle of Hooghoudt (1940), the ground water near the drains flows radially towards them. In the area of radial flow, the cross-section of the flow at a distance X from the drains is formed by the circumference of a quarter circle with a length ½πX. This principle is conceptualized in *Figure B* by letting an imaginary impermeable layer slope away from the centre of the drain at an angle with a tangent ½π.

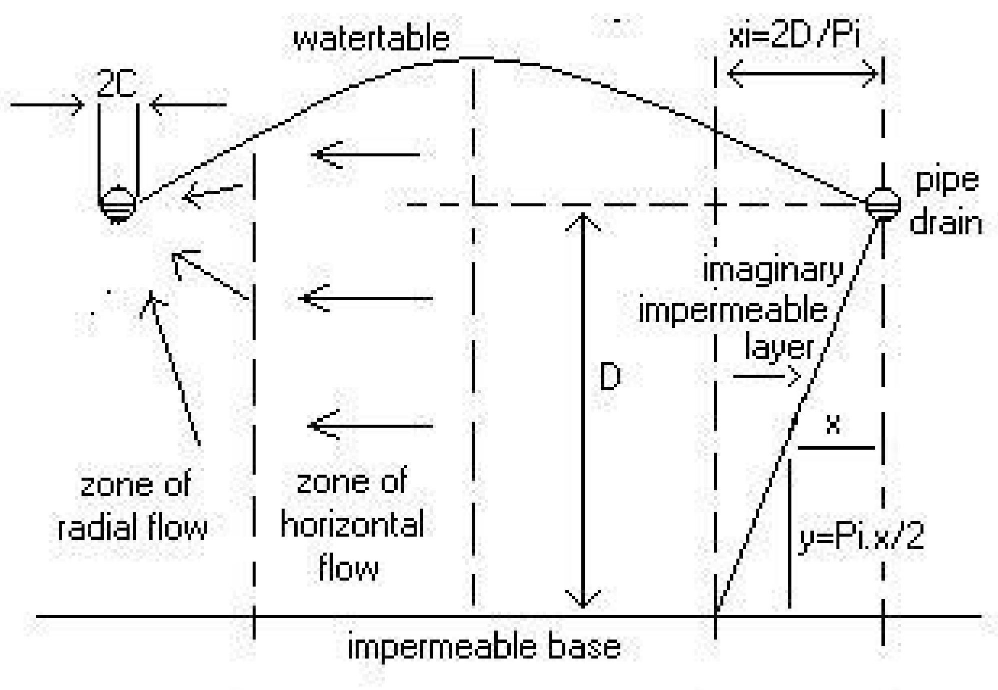


Figure B. Vertically two-dimensional flow of ground water to parallel pipe drains placed at equal depth in a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation.

The depth of the imaginary sloping layer at distance X, taken with respect to the centre point of the drain, equals Y = ½πX (m), so that the vertical cross-section of the flow is equal to that of the quarter circle. At the drain, where X = C, the depth Y equals $Yc = \frac{1}{2}\pi C$, which corresponds to a quarter of the drain's circumference.

The sloping imaginary layer intersects the real impermeable base at the distance:

$$W = 2D/\pi \tag{11}$$

The area of radial flow is found between the distances X=C and X=W. Beyond distance X=W, the vertical cross-section equals Y=D.

To include the flow approaching the drain from above the drain level, the total vertical cross-section in the area of radial flow is taken as J=Y+Hx.

The horizontal component Vx of the flow velocity in the vertical section is taken constant, but its vertical component need not be constant. Now, Equation 10 can be written for two cases as:

$$C<X<W: \quad Hx = \int_C^X \left[\frac{R(N-X)}{K(Hx+\frac{1}{2}\pi X)}\right] dX - \int_C^X \left[\frac{Hn-Hx}{N-X}\right] dX$$

$$W<X<N: \quad Hx = \int_C^X \left[\frac{R(N-X)}{K(Hx+D)}\right] dX - \int_C^X \left[\frac{Hn-Hx}{N-X}\right] dX$$

The second term in the above equations represents the energy associated with the incoming recharge.

These are the equations used in section *1.B The simulative procedure*