RAINFALL-RUNOFF RELATIONS OF A SMALL VALLEY ASSESSED WITH A NON-LINEAR RESERVOIR MODEL

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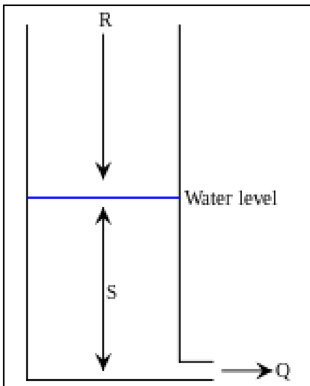
Abstract: - A non-linear reservoir model is used to analyze the rainfall-runoff relations in a small valley (watershed, hydrological catchment), in Sierra Leone. The concept of a linear reservoir, which uses a constant reaction factor, for use in hydrological modeling is well known but often not effective. Non-linear reservoirs, having reaction factors that increase with increasing water storage, are less frequently applied but they have more promise. One may use reaction factors that are a linear function of the storage, which implies that the reservoir reacts quicker to rainfall under wet than under dry conditions. The reaction factor could also be a quadratic function of the storage so that the discharge increases progressively with increasing water storage. The characteristic functions of the reaction factors of the catchments are first found by calibrating part of the data, and thereafter they are verified with the remaining data. In the case study, the calibrations were done with a high precision. The verification (validation), however, was complicated by the fact that the valley bottom was used for rice cultivation and that the farmers interfered in the natural runoff process so that the reservoir characteristics changed in time. Yet, the non-linear reservoir model could be verified reasonably well finding the reservoir function by calibration over a 10-day period and applying that function to a two-day period of peak discharges within the selected 10 days. Hence, the non-linear reservoir model has proved to be effective. Free software for non-linear reservoir models is available.

Keywords: rainfall-runoff relations, small valley, rice cropping, non-linear reservoir model

1 Introduction, Reservoir Models

The linear reservoir (LR) is described by D.A.Kraijenhoff van de Leur [Ref. 1] and its principles are given in figure 1.



R = recharge from rainfall {dimension L/T}

S = storage {dimension L}

Q = discharge, runoff {dimension L/T}

Fig.1. The concept of a linear reservoir (LR).

For the linear reservoir the following equations hold:

LR reservoir function:

$$Q = \alpha . S \qquad \text{(Eq. 1)}$$

where α = reaction factor {dimension 1/T}

Differentiating S to time T gives

$$dS/dT = d(Q/\alpha)/dT = R - Q \qquad \text{(Eq. 2)}$$

Integrating Eq. 2 with limits Q_i, Q_{i+1}, T_i and T_{i+1} yields:

$$Q_{i+1} = Q_i \exp\{-\alpha (T_{i+1} - T_i)\} + R_i [1 - \exp\{-\alpha (T_{i+1} - T_i)\}] \qquad \text{(Eq. 3)}$$

where Q_i and Q_{i+1} are Q at time T_i and T_{i+1} respectively ($T_{i+1} > T_i$) and R_i is the recharge from time T_i to T_{i+1}. Here, i is the serial number of the time steps, i = 1, 2, 3 . . . . n (n for the last step).

With Equation 3 the discharge Q_{i+1} can be calculated from R_i , Q_i , α and the time difference.

The instantaneous unit hydrograph (IUH) method, which is also used in rainfall-runoff relations, is found from the first part of Eq. 3 as :

$IUH_i = Q_u \exp\{-\alpha(T - T_{i+1})\}$ for $T > T_{i+1}$ where

$Q_u = R_i [1 - \exp\{-\alpha(T_{i+1} - T_i)\}]$ which is found from the second part.

To obtain the total hydrograph over the time sequence studied, the respective partial IUH's are superimposed (example in figure 2).

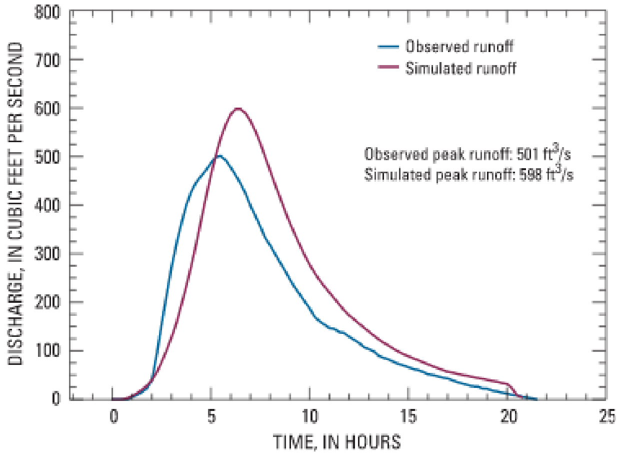


Fig.2. Simulation by means of the IUH method of discharge runoff at Mallard Creek near Harrisburg, North Carolina, for the storm of December 12, 1996 [Ref. 2].

The IUH method has similarity with the LR model. However the determination of α is cumbersome because when $R_i = 0 \rightarrow Q_m = 0 \rightarrow IUH_i = 0$, whereas for the LR model, that works integrated over time instead of by instantaneous parts (units), periods with $R_i = 0$ are useful, as explained in continuation.

In the LR model, when $R_i = 0$ (no recharge), equation 3 reduces to:

$$Q_{i+1} = Q_i \exp \{-\alpha (T_{i+1} - T_i)\} \quad \text{(Eq. 4)}$$

This equation gives the possibility to determine α from Q_i, Q_{i+1} and $T_{i+1} - T_i$ during a dry spell:

$$\alpha = -\ln (Q_{i+1}/Q_i)/(T_{i+1} - T_i) \quad \text{(Eq. 5)}$$

Nevertheless, the LR concept is often too simple to characterize the watershed as its reaction factor is usually more complicated. Therefore Nash [Ref. 3] employed a cascade of linear reservoirs, one reservoir emptying into the next, while Kraijenhoff [Ref. 1] used a number of parallel reservoirs over which the rainfall is distributed in some proportion, while the reservoirs joined their discharge.

In hydrological practice, the concept of non-linear reservoirs has seldom been applied. Instead of a number of reservoirs with a constant reaction factor, one could employ one non-linear reservoir with a reaction factor that changes linearly with storage (figure 3) instead of being a constant, thus avoiding the difficulty of dealing with multiple reservoirs.

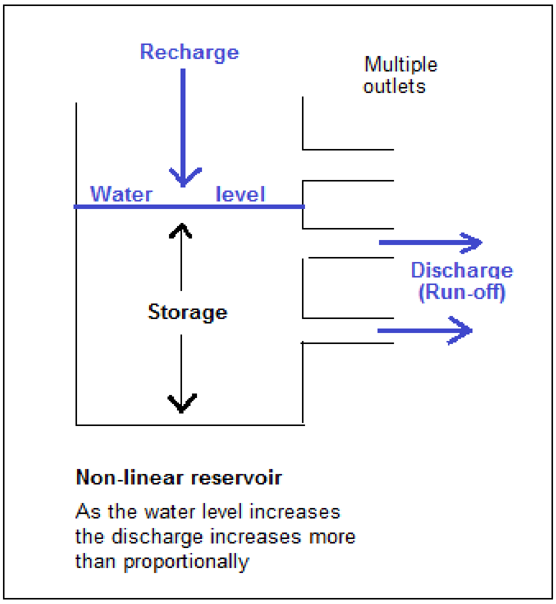


Fig.3. A non-linear reservoir with multiple outlets whereby the discharge increases more than proportionally with increasing storage.

The reaction factor is now written as

$$\alpha_i = B.Q_i + C \quad \text{(Eq. 1a)}$$

which can be used in equation 3 to calculate the runoff Q.

The RainOffT software [Ref. 4] solves this numerically and optimizes the values of A and B so that a maximum fit is obtained of the measured Q values to the calculated ones according to the model.

The program also permits to go a step further using a reservoir consisting of two parts (figure. 4)

The original RainOff software [Ref. 3] solves the model described in figure 4 using the equivalent of equation 1a:

$$\alpha_{1i} = B_1.Q_i + C_1 \quad [Q_i < Q_Z] \quad \text{(Eq. 1b)}$$

$$\alpha_{2i} = B_2.Q_i + C_2 \quad [Q_i > Q_Z] \quad \text{(Eq. 1c)}$$

where Q_Z is the runoff divide, i.e. the runoff when the lower part of the reservoir is just full and the upper part is empty. Equation 5 is used to determine the values of α_{1i} and α_{2i} reckoning with the Q conditions given above.

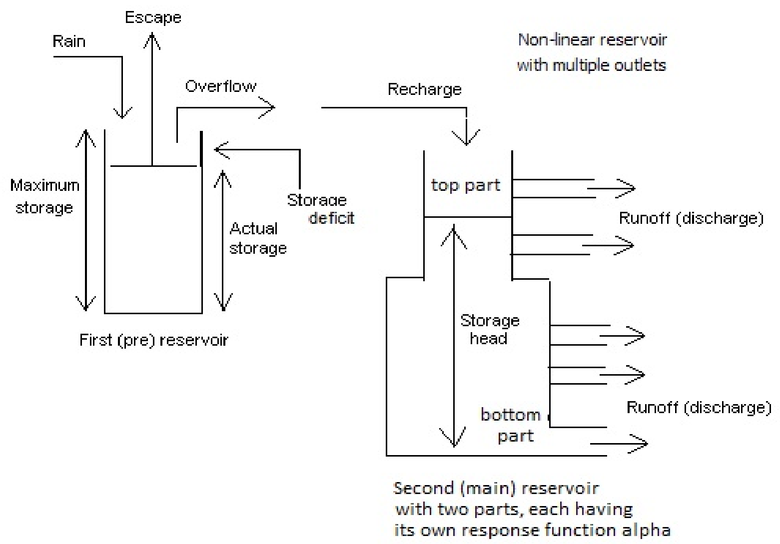


Fig. 4. A reservoir model with the main reservoir composed of two parts, each with multiple outlets, while in the top part the storage level, given a unit of rainfall, increases more rapidly than in the lower part.

The second (main) reservoir is preceded by a pre-reservoir that provides the net recharge (effective precipitation) after deducting evaporation (escape) and replenishment of the soil moisture from the rainfall.

Figure 5 gives an example of the α_{1i} and α_{2i} (reservoir functions) obtained from regressions of calculated α values according to 6a and b during dry spells on discharge. The separation point here is Q_Z = 1.15 mm/hr.

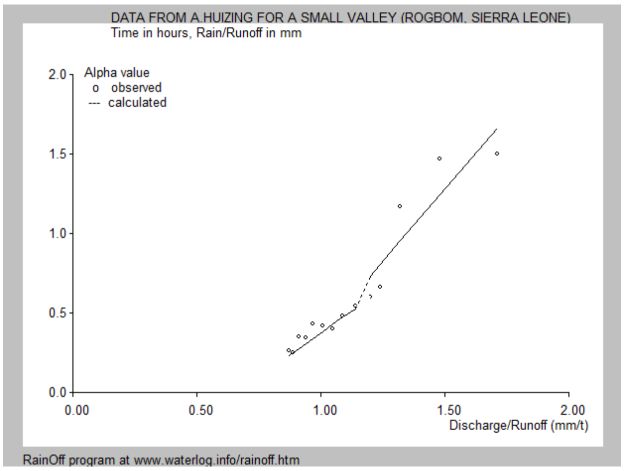


Fig.5. Example of two reservoir functions below and above a separation point Q_Z=1.15 mm/hr

When the main reservoir consists of a container with, towards the top, linearly inward sloping walls, then one obtains a reservoir reaction factor (α_i) that is a quadratic function of Q_i:

$$\alpha_i = A.Q_i^2 + B.Q_i + C \qquad \text{(Eq. 1d)}$$

which can be used in equation 3 to calculate the runoff Q.

The software program RainOffQ [Ref. 3] solves this case numerically and optimizes the values of A, B and C so that a maximum fit is obtained of measured Q values with calculated ones according to the model.

An overview of the rainfall-runoff relations equivalent to equation 3 for the linear reservoir but adjusted to non-linear reservoirs is shown in the appendix.

2 Application to a valley in Sierra Leone

Gunneweg et al. [Ref. 5] give description of the hydrologic situation and water management systems of small valleys in W. Africa. Figure 6 sketches some of these characteristics and figure 7 gives a picture of a small valley with rice cultivation.

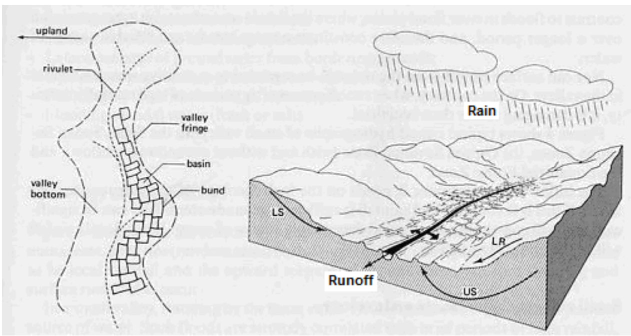


Fig.6. Sketch of physical and hydrological characteristics of a small valley with rice fields in Sierra Leone [Ref. 5]

Fig.7. A small valley with rice cultivation in W. Africa. The central drain is temporarily overgrown. [Ref. 5]

Huizing [Ref. 6] collected hourly rainfall-runoff relations in a small cultivated valley (Rogbom) in Sierra Leone near the township Makenni, during the months of July and August 1987. Measurements were made on 6 days spaced apart and two continuous periods of 10 days.

Of the 6 separate days there were 5 with considerable rainfall in the morning followed by a dry afternoon.

These days are July 13 and 20, August 6 and 24. and 17 September.

Figure 8 shows the analysis, using the RainOffT version, for August 6, which day was selected because there were two rainy spells, while the other days only had only one. The coefficient of explanation (96%) is quite high. The reaction factor is found as

$$\alpha_i = 0.185\ Q_i\ - 0.176.$$

Figure 9 confirms that the runoff increases more quickly at higher rainfalls so that a non-linear reservoir is appropriate.

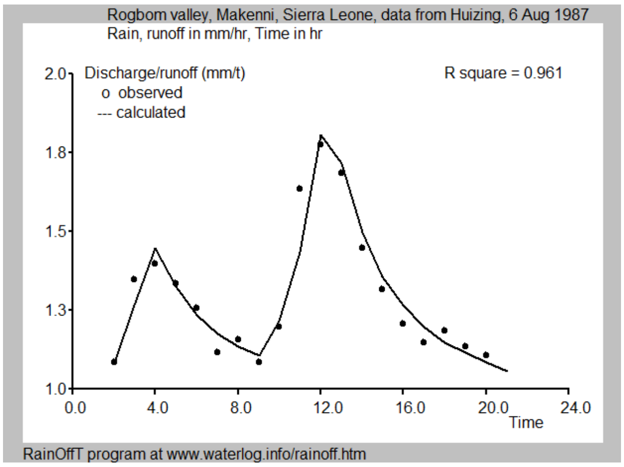


Fig.8. RainOff results: calculated and observed hourly runoff, Rogbom valley, August 6, 1987.

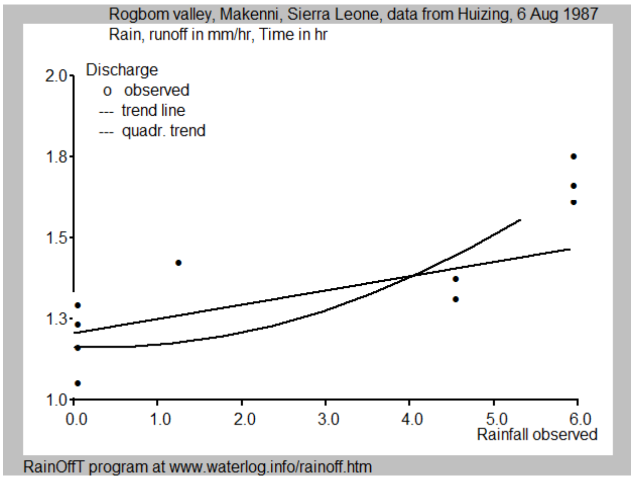


Fig.9. RainOff results: observed hourly rain-fall and runoff, Rogbom valley, August 6, 1987.

The quadratic up-curving trend in figure 9 suggests that the runoff increases more rapidly at higher rainfalls as is expected by the RainOff non-linear reservoir model.

Figure 10 depicts the runoff simulation for a 10-day period (17-27 August 1987).

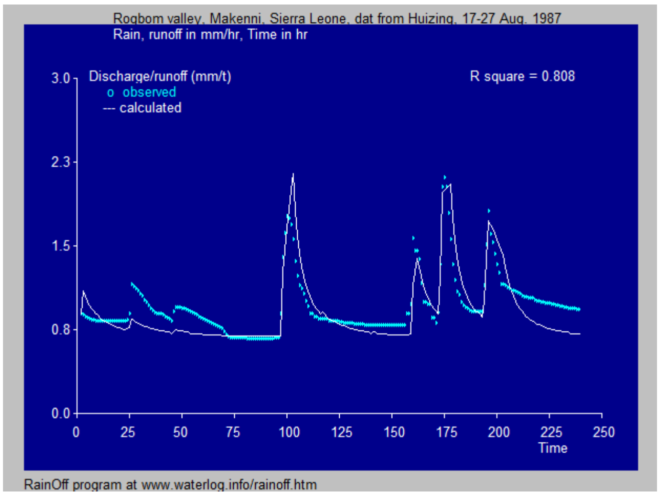


Fig.10. RainOff results: calculated and observed hourly runoff, Rogbom valley, period 17–27 August 1987.

In Figure 10 the fit of the model to the data is not perfect, the coefficient of determination is relatively low (0.81 or 81%). For this there are two reasons:

1 – There are many days (107) with constant runoff during dry spells , while the model expects the runoff to decrease gradually during such periods. In the valley bottom rice is cultivated, which makes that the runoff is influenced by farmers and that the runoff conditions are not entirely natural. RainOff, on the other hand, assumes natural conditions without interference by mankind during the runoff process.

2 – The trend of observed runoff versus rainfall is one of a gradually smaller runoff increase with increasing rainfalls (figure 11). This is the opposite of the trend shown in figure 7, and it is not in accordance with the assumptions made for the non-linear model.

For both these reasons, the RunOff software is not able to handle the rainfall-runoff relation over a longer period adequately. These two adverse features are also not in accordance to the generally accepted hydrological assumptions.

From figure 10, it appears that day 21 August (starting at 96 hrs) and days 24 and 25 August (from hour 168 to hour 188) would be suitable to apply the model as these periods have higher discharges.

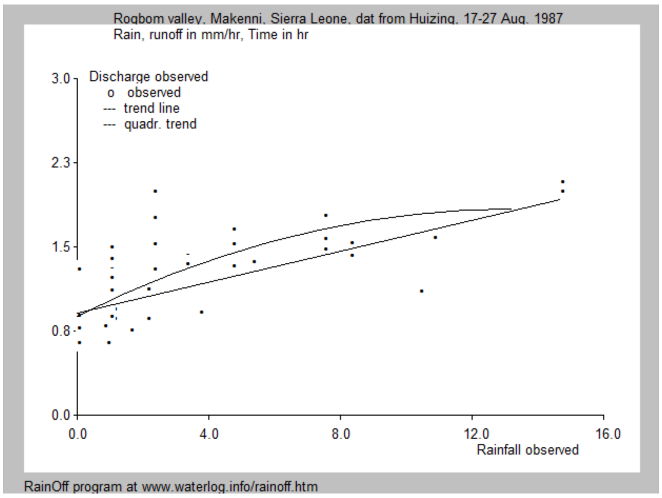


Fig.11. RainOff results: observed hourly rainfall and runoff, Rogbom valley, period 17–27 August 1987.

On this basis, the period of 23-24 August was selected for closer inspection because there are two runoff peaks and there is no period with constant runoff during several days.

The excellent data fit to the runoff model for this period is shown in figure. 12. The coefficient of determination is quite high (96%).

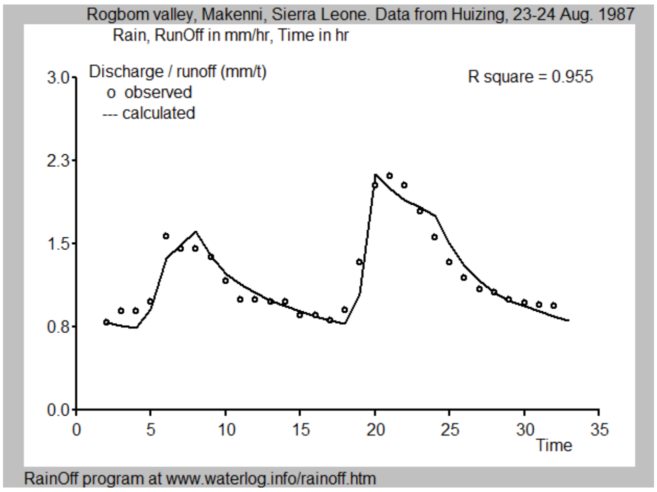


Fig. 12. RainOff results: observed hourly rainfall and runoff, Rogbom valley, period 23 – 24 August.

3 Verification (validation)

For verification it is required to use the parameters of the reservoir model obtained during the calibration rounds and apply these to a data set that was not analyzed before. However, the parameters for the examples given before produced large differences amongst the different examples as shown in table 1.

Table 1. Parameters of the reservoir reaction factor ($\alpha_i = B.Q_i + C$) for the RainOff model employing a reservoir consisting of two parts (Fig. 3)

Date of data	Runoff separation point (Q_Z, mm/hr)	Runoff (Q_i) Condition	B coefficient	C value
6 August	1.07	$Q_i > Q_Z$	0.1887	− 0.1525
		$Q_i < Q_Z$	0.0000	0.0409
23-24 August	0.98	$Q_i > Q_Z$	0.1145	− 0.0266
		$Q_i < Q_Z$	0.1305	− 0.0366
17 – 27 August	1.31	$Q_i > Q_Z$	0.0000	0.0584
		$Q_i < Q_Z$	0.1193	− 0.0771

Apparently, the environmental conditions change strongly from time to time, which prevents the use of a standard set of parameters for all the months of the summer period studied (July, August and September). Possibly, one reason for this variation is the vegetative development of the rice crop cultivated in the bottom of the valley, together with heightening and strengthening of the bunds around the rice fields when the crop grows bigger, as well as diverting runoff water for irrigation. Yet, verification can be done within the month.

In figure 13 a screen shot of the input tab sheet of the RainOff program, the parameters A_1, B_1, A_2 and B_2 of the reaction factors $\alpha_{1i}=B_1.Q_i+C_1$ and $\alpha_{2i}=B_2.Q_i+C_2$ plus the dividing point Q_Z found for the 10-day period 17–27 August (Fig. 8, Table 1, Fig. 10), were used to simulate the runoff from the rainfall data for the 2-day period of 22–34 August which includes the peak discharges of the 10-day period (Fig. 10, Fig.12).

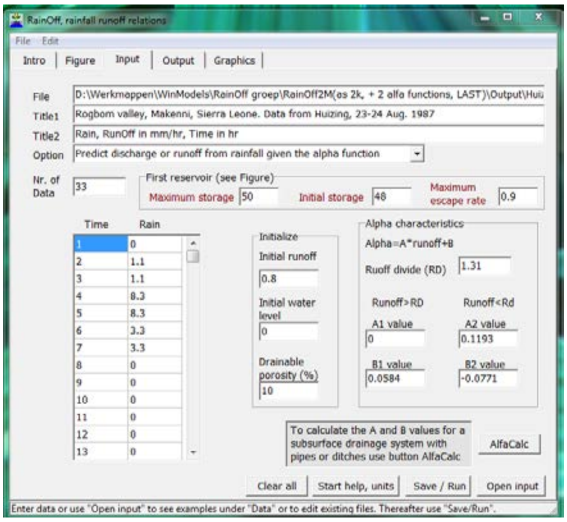


Fig.13. Screen-print of input tab sheet of the RainOff program showing the data for runoff simulation of period 23-24 August.

In figure 13 the characteristics of the reservoir function α (in the figure called Alpha) derived from the period of 17-27 August as shown in table 1 have been entered.

The results of the simulation are shown in figure 14 and compared with the measured runoff. The agreement, though not perfect, is reasonable, taking into account the environmental changes that may have occurred.

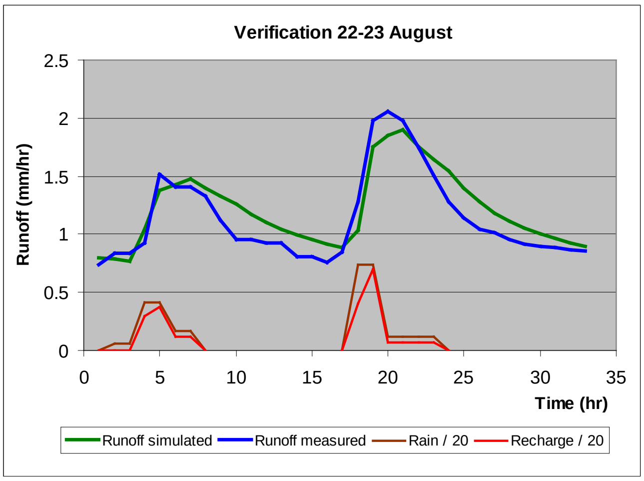


Fig.14. The runoff hydrograph for the period of 22-23 August (blue line, see also Fig. 12) is simulated (green line) using the parameters of the reservoir function α derived for the period of 17–27August (see Fig. 10 and Table 1).

4 Conclusions

The RainOff model has produced reliable results in short term (1 or 2 day) simulations (figures 8 and 12). This leads to the conclusion that the software is valuable. The parameters of the reservoir functions, however, were quite different from month to month due to changing environmental conditions, rice cultivation practices and human interference with the runoff processes in the valley. Verification between months, therefore, is not practical. However, verification within the month of August has produced an acceptable agreement of simulated and measured runoffs.

The results of the simulations have proved that the non-linear reservoir model, using reaction factors that increase with increasing storage, perform better than the well-known linear reservoir model in which the reaction factor is a constant. This confirms the notion that a unit amount of recharge discharges more than proportionally faster as the watershed is wetter.

Also, use of the non-linear reservoir model avoids the complicated undertaking of handling a series of linear reservoirs. Free software for rainfall-runoff relations modeled with non-linear reservoirs is available [Ref. 4].

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APPENDIX

(Summary of rainfall−runoff equations)

1 - Linear reservoir with constant reaction factor (Fig. 1):

$$Q_{i+1} = Q_i \exp\{-\alpha\,(T_{i+1}-T_i)\} + R_i\,[1-\exp\{-\alpha\,(T_{i+1}-T_i)\}] \qquad \text{(Eq. 3)}$$

where Q is the runoff (discharge), R_i is the recharge (effective rainfall from time T_i to T_{i+1}), α is the constant reaction factor, Q_i and Q_{i+1} are Q at time T_i and T_{i+1} respectively ($T_{i+1} > T_i$). Here, i is the serial number of the time steps, i = 1, 2, 3, ... n.

2 - Non-linear reservoir with linear reaction factor (Fig. 3):

$$Q_{i+1} = Q_i \exp\{-(B.Q_i + C).(T_{i+1}-T_i)\} + R_i\,[1-\exp\{-(B.Q_i + C).(T_{i+1}-T_i)\}]$$

(Eq. 3a)

so that $\alpha_i = B.Q_i + C$. The factor B and constant C are to be found by optimization using an iterative algorithm as is done in the RainOffT program [Ref.4].

3 – Composite non-linear reservoir, each part with linear reaction factor (Fig. 4):

3a - if $[Q_i<Q_Z]$ then

$$Q_{i+1} = Q_i \exp\{-(B_1.Q_i + C_1).(T_{i+1}-T_i)\} + R_i\,[1-\exp\{-(B_1.Q_i + C_1).(T_{i+1}-T_i)\}]$$

(Eq. 3c)

3b - if $[Q_i>Q_Z]$ then

$$Q_{i+1} = Q \exp\{-(B_2.Q_i + C_2).(T_{i+1}-T_i)\} + R_i\,[1-\exp\{-(B_2.Q_i + C_2).(T_{i+1}-T_i)\}]$$

(Eq. 3d)

where Q_Z is the separation point for the lower and higher discharge ranges, while B_1, C_1, B_2, and C_2, are the factors B and constants C in the different Q ranges.

The standard RainOff software [Ref. 4] follows this procedure (Fig. 5) obtaining the B and C values, for the periods in which $R_i = 0$, from linear regressions on the basis of the equations :

$$Q_i = B_1\{-\ln(Q_{i+1}/Q_i)/(T_{i+1}-T_i)\}+C_1 \quad \text{when } Q_i<Q_Z$$
$$Q_i = B_2\{-\ln(Q_{i+1}/Q_i)/(T_{i+1}-T_i)\}+C_2 \quad \text{when } Q_i>Q_Z$$

while the value of Q_Z is found by optimization using an iterative algorithm.

4 - Non-linear reservoir with quadratic reaction factor:

$$Q_{i+1} = Q_i \exp\{-(A.Q_i^2 + B.Q_i + C).(T_{i+1}-T_1)\} + R_i\,[1-\exp\{-(A.Q_i^2 + B.Q_i + C).(T_{i+1}-T_i)\}]$$

(Eq. 3e)

Here $\alpha_i = A.Q_i^2 + B.Q_i + C$. The factors A and B and the constant C can be determined by optimization using an iterative algorithm, as is done with the RainOffQ software [Ref. 4]. Since in this case three parameters have to be optimized instead of two, the calculation procedure is time consuming.