THE FULL ENERGY BALANCE OF GROUNDWATER FLOW APPLIED TO SUBSURFACE LAND DRAINAGE IN (AN)ISOTROPIC SOILS BY PIPES/DITCHES WITH/WITHOUT ENTRANCE RESISTANCE

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Abstract: In agricultural land drainage the depth of the water table in relation to crop production and the groundwater flow to drains plays an important role. The full energy balance of groundwater flow, equivalent to Joule's law in electricity, developed by Boonstra and Rao (1994), and used for the groundwater flow in unconfined aquifers, can be applied to subsurface drainage by pipes or ditches with the possibility to introduce entrance resistance and/or (layered) soils with anisotropic hydraulic conductivities. Owing to the energy associated with the recharge by downward percolating water, it is found that use of the full energy balance leads to lower water table elevations than the classical methods employing the Darcy equation. The full energy balance cannot be solved analytically and a computerized numerical method is needed. An advantage of the numerical method is that the shape of the water table can be described, which was possible with the traditional methods only in exceptional situations, like drains without entrance resistance, resting on an impermeable layer in isotropic soils. The software package EnDrain has been developed to deal with the full energy balance and it is used to analyze a variety of drainage conditions.

Key words: Subsurface land drainage, groundwater flow, full energy balance, anisotropic hydraulic conductivity, enattrance resistance

1 Introduction, ditch drains

Agricultural land drainage is widely practiced in the world, but it needs to be applied with care [Ref. 1]. Crop yields are reduced when the water table is too shallow (Ref. 2, figure 1).

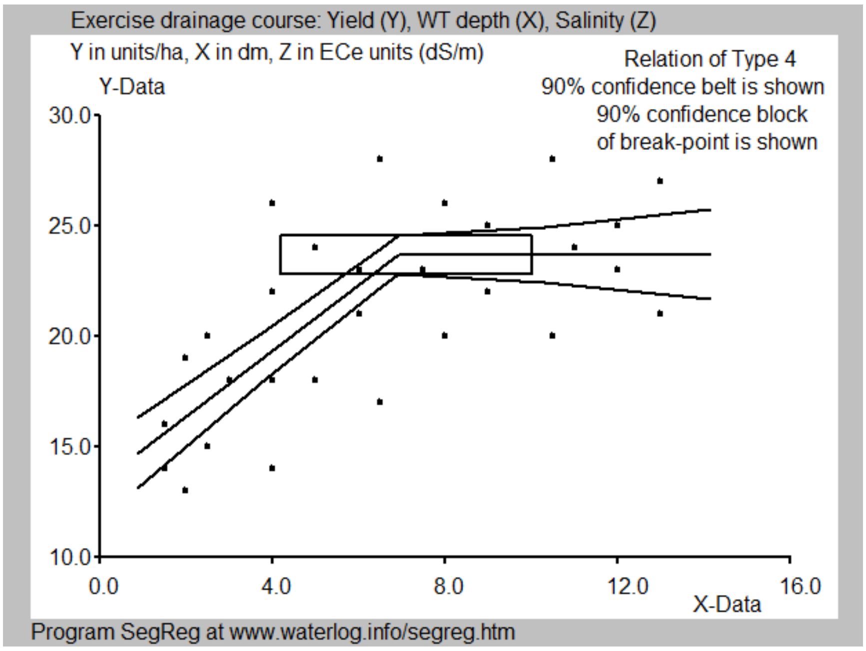

Fig. 1 Statistical analysis of the relation between crop yield (Y) and seasonal average depth of the water table (X in dm). When the water table is shallower than 8 dm the yields decline.

The depth of the water table depends on the drain distance (spacing). Spacing calculations between consecutive lateral drains are closely related to water flow towards the drains [Ref. 3].

Boonstra and Rao [Ref. 4], therefore, introduced the complete energy balance of groundwater flow. It is based on equating the change of hydraulic energy flux over a horizontal distance to the conversion rate of hydraulic energy into to friction of flow over that distance. The energy flux is calculated on the basis of a multiplication of the hydraulic potential and the flow velocity, integrated over the total flow depth. The conversion rate is determined in analogy to the heat loss equation of an electric current, named after Joule. For the differentiation of the integral equation, Leibniz's integral rule had to be applied.

Assuming (1) steady state fluxes, i.e. no water and associated energy is stored, (2) vertically two-dimensional flow, i.e. the flow pattern repeats itself in parallel vertical planes, (3) the horizontal component of the flow is constant in a vertical cross-section, and (4) the soil's hydraulic conductivity is constant from place to place, it was found that [Ref. 4]::

$$\frac{dJ}{dX} = \frac{Vx}{Kx} - \frac{R(J-Jr)}{Vx.J} \qquad \text{(Eq. 1)}$$

where: J is the level of the water table at distance X, taken with respect to the level of the impermeable base of the aquifer (m), Jr is a reference value of level J (m), X is a distance in horizontal direction (m), Vx is the apparent flow velocity at X in horizontal X-direction (m/day), Kx is the horizontal hydraulic conductivity (m/day), R is the steady recharge by downward percolating water stemming from rain or irrigation water (m/day), dX is a small increment of distance X (m), dJ is the increment of level J over increment dX (m), dJ/dX is the gradient of the water table at X (m/m).

The last term of equation 1 represents the energy associated with the recharge R. When the recharge R is zero, Equation 1 yields Darcy's equation, which does not account for the complete energy balance. The negative sign before Vx indicates that the flow is positive when the gradient dJ/dX is negative, i.e. the flow follows the descending gradient, and vice versa.

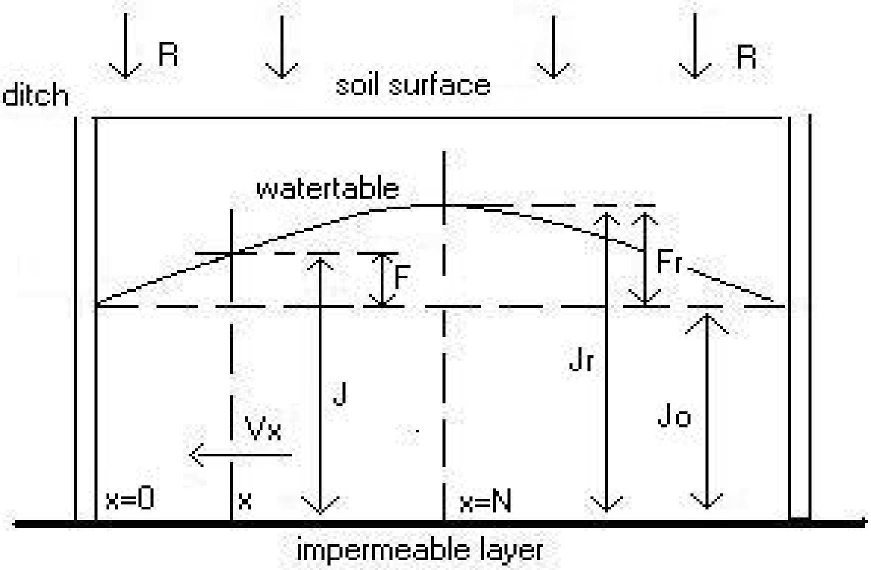

Fig. 2. Vertically two-dimensional flow of ground water to parallel ditches resting on the impermeable base of a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation.

Figure 2 shows the vertically two-dimensional flow of ground water to parallel ditches resting on a horizontal impermeable base of a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation (R>0, m/day). At the distance X=N (m), i.e. midway between the ditches, there is a water divide. Here the water table is horizontal. At the distance X≤N, the discharge of the aquifer equals Q = −R(N−X) (m^2/day) where the minus sign indicates that the flow is contrary to the X direction. From this water balance we find Vx = Q/J = −R(N−X)/J (m/day).

With this expression for the velocity Vx, Equation 1 can be changed into:

$$\frac{dJ}{dX} = \frac{R(N-X)}{Kx.J} - \frac{Jr-J}{N-X} \qquad \text{(Eq. 2)}$$

Setting F = J−Jo, and Fr = Jr−J, where Jo is the value of J at X=0, i.e. at the edge of the ditch, it is seen that F represents the level of the water table with respect to the water level in the ditch (the drainage level). Applying the condition that dF/dX=0 at X=N, we find from Equation 2 that Fr=Fn, where Fn is the value of F at X=N, and:

$$\frac{F}{X} = \frac{R(N-X)}{Kx.J} - \frac{Fn-F}{N-X} \qquad \text{(Eq. 3)}$$

Introducing the drain radius C (m), and integrating equation 3 from X=C to any other value X, gives:

$$F = \int_C^X \frac{R(N-X)}{Kx.J} dX - \int_C^X \frac{Fn-F}{N-X} dX \qquad \text{(Eq. 4)}$$

Integration of the last term in equation 4 requires advance knowledge of the level Fn. To overcome this problem, a numerical solution and a trial and error procedure must be sought. Oosterbaan et al. gave a method of numerical solution and an example from which it was found that the water table is lower than calculated according to the traditional method, except at the place of the ditch.

In the following sections, the equations will be adjusted for calculating subsurface drainage with pipe drains or ditches that do not penetrate to the impermeable base, while entrance resistance may occur and the soil's hydraulic conductivity may be anisotropic.

2 Pipe (tube, tile) drains

Figure 3 shows the vertically two-dimensional flow of ground water to parallel pipe drains with a radius C (m), placed at equal depth in a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation (R>0, m/day). The impermeable base is taken horizontal with a depth D>C (m) below the center point of the drains. At the distance X=N (m), i.e. midway between the drains, there is a water divide. Here the water table is horizontal.

We consider only the radial flow approaching the drain at one side, because the flow at the other side is symmetrical, and also only the flow approaching the drain from below drain level.

According to the principle of Hooghoudt [Ref. 5], the ground water near the drains flows radially towards them. In the area of radial flow, the cross-section of the flow at a distance X from the drains is formed by the circumference of a quarter circle with a length ½πX. This principle is conceptualized in figure 3 by letting an imaginary impermeable layer slope away from the center of the drain at an angle with a tangent ½π.

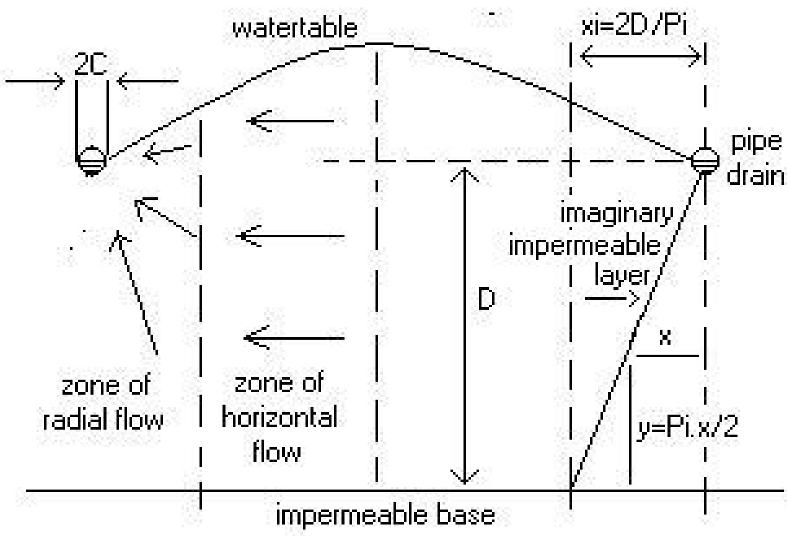

Fig. 3. Vertically two-dimensional flow of ground water to parallel pipe drains placed at equal depth in a phreatic aquifer recharged by evenly distributed percolation from rainfall or irrigation.

The depth of the imaginary sloping layer at distance X, taken with respect to the center point of the drain, equals $Y = \frac{1}{2}\pi X$ (m), so that the vertical cross-section of the flow is equal to that of the quarter circle. At the drain, where X = C, the depth Y equals $Y_c = \frac{1}{2}\pi C$, which corresponds to a quarter of the drain's circumference.

The sloping imaginary layer intersects the real impermeable base at the distance:

$$Xi = 2D / \pi \qquad \text{(Eq. 5)}$$

The area of radial flow is found between the distances X=C and X= Xi. Beyond distance X= Xi, the vertical cross-section equals Y = D.

To include the flow approaching the drain from above the drain level, the total vertical cross-section in the area of radial flow is taken as J = Y+F.

The horizontal component Vx of the flow velocity in the vertical section is taken constant, but its vertical component need not be constant. Now, Equation 4 can be written for two cases as:

if C<X<Xi:

$$F = \int_C^X \frac{R(N-X)}{Kx(F+\frac{1}{2}\pi X)}\,dX - \int_C^X \frac{Fn-F}{N-X}\,dX \qquad \text{(Eq. 6a)}$$

if Xi<X<N:

$$F = \int_C^X \frac{R(N-X)}{Kx(F+D)}\,dX - \int_C^X \frac{Fn-F}{N-X}\,dX \qquad \text{(Eq. 6b)}$$

3 NUMERICAL INTEGRATION

For the numerical integration, the horizontal distance N is divided into a number (T) of equally small elements with length U, so that U = N / T. The elements are numbered S = 1, 2, 3, ..., T.

The height F at a distance defined by the largest value of distance X in element S, is denoted as F_S. The change of height F over the S-th element is denoted as G_S, and found from:

$$G_S = F_S - F_{S-1}$$

The average value of height F over the S-th element is:

$$\underline{F}_S = F_{S-1} + \tfrac{1}{2}G_{S-1}$$

For the first step (S=i, see Equation 10 below), the value of $\underline{F}_S = \underline{F}_i$ must be determined by trial and error because then the slope $G_{S-1} = G_{i-1}$ is not known.

The average value of the horizontal distance X of the S-th element is found as:

$$\underline{X}_S = U(S-0.5)$$

The average value of depth Y over the S-th element is:

$$\underline{Y}_S = \tfrac{1}{2}\pi\underline{X}_S \quad \text{when} \quad C<\underline{X}_S<Xi \qquad \text{(Eq. 7a)}$$

$$\underline{Y}_S = D \quad \text{when} \quad Xi<\underline{X}_S<N \qquad \text{(Eq. 7b)}$$

Equation 3 can now be approximated by:

$$G_S = U(A_S+B_S) \qquad \text{(Eq. 8)}$$

where:

$$A_S = R(N-\underline{X}_S) / Z_S$$

with:

$$Z_S = Kx(\underline{Y}_S+\underline{F}_S) \quad \text{when} \quad C<\underline{X}_S<Xi \qquad \text{(Eq. 9a)}$$

$$Z_S = Kx(D+\underline{F}_S) \quad \text{when} \quad Xi<\underline{X}_S<N \qquad \text{(Eq. 9b)}$$

and:

$$B_S = (\underline{F}_S-F_T) / (N-\underline{X}_S)$$

where F_T is the value of $\underline{F}_S$ when S = T. The factor Z_S can be called transmissivity (m^2/day) of the aquifer.

Now, the height of the water table at any distance X can be found, conform to Equations 6a and 6b, from:

$$F_S = \sum_{i}^{S} G_S \qquad \text{(Eq. 10)}$$

where i is the initial value of the summations, found as the integer value of:

$$i = 1 + C / U \qquad \text{(Eq. 11)}$$

so that the summation starts at the outside of the drain.

Since F_S depends on B_S and B_S on F_S and F_T, which is not known in advance, Equations 8 and 10 must be solved by iterations.

Omitting the last terms of Equations 6a and 6b, i.e. ignoring part of the energy balance, and further in similarity to the above procedure, a value G_S* can be found as:

$$G_S^* = R.U(N-\underline{X}_S)/Z_S^* \qquad \text{(Eq. 12)}$$

where:

$$Z_S^* = Kx(\underline{Y}_S+\underline{F}_S^*) \quad \text{when} \quad C<\underline{X}_S<Xi$$

$$Z_S^* = Kx(D+\underline{F}_S^*) \quad \text{when} \quad Xi<\underline{X}_S<N$$

and:

$$\underline{F}_S^* = F_{S-1}^*+\tfrac{1}{2}G_{S-1}^*$$

Thus the height of the water table, in conformity to Equation 10, is:

$$F_S^* = \sum_{i}^{S} G_S^* \qquad \text{(Eq. 13)}$$

This equation corresponds to the classical Hooghoudt equation [Ref. 5] and will be used for comparison with Equation 10 (accounting for the energy balance).

4 Example of a numerical solution

To illustrate the numerical solutions we use the same data as in an example of drain spacing calculation with Hooghoudt's equation given by Ritzema [Ref. 6]:

N = 32.5 m	C = 0.1 m
Kx = 0.14 m/day	R = 0.001 m/day
D = 4.8 m	Fn* = 1.0 m

The calculations for the numerical solutions were made on a computer with the EnDrain program [Ref. 6]. The results are presented in table 1 and in figure 4.

Table 1 gives the values of height F_S* (no energy balance), water table gradient G_S*/U, height F_S (with energy balance) and gradients G_S/U, A_S and B_S at some selected values of distance X with steps of U = 0.05 m, so that in total 650 steps are taken with a large number of iterations. Smaller values of step U do not yield significantly different results.

Table 1. Shape of the water table ignoring the energy balance (F*) and accounting for the energy balance (F) as calculated with EnDrain for the conditions described in the example of Section 4, using equations 8 and 10 with steps U = 0.05 m.

Distance from drain center (m)	Height of water table F* (m) (Darcy)	Gradient of F* G / U (m/m)	Height of water table F (m) (full energy balance)	Gradient needed for flow (m/m)	Adjustment of F* due to energy of recharge (m/m)
0.75	0.24	0.164	0.23	0.147	–0.0169
1.5	0.33	0.085	0.31	0.070	–0.0149
3	0.42	0.042	0.37	0.029	–0.0134
6	0.53	0.037	0.45	0.025	–0.0119
9	0.63	0.032	0.52	0.022	–0.0104
12	0.72	0.028	0.58	0.019	–0.0092
15	0.80	0.024	0.64	0.016	–0.0078
18	0.86	0.019	0.68	0.013	–0.0064
21	0.91	0.015	0.71	0.010	–0.0052
24	0.95	0.012	0.74	0.008	–0.0039
27	0.98	0.007	0.76	0.005	–0.0024
30	0.99	0.004	0.77	0.0025	–0.0015
33	1.00	0	0.78	0	0

It is seen from table 1 that the Fn* value (i.e the value of F* at X = N = 33 m) equals 1.00 m. This is in agreement with the value used by Ritzema [Ref. 5].

In table 1 it can also be seen that the height of the water table above drain level midway between the drains (F*) is 1.00 m in case of ignoring part of the water balance whereas it is only 0.78 m when accounting for the whole water balance. The Hooghoudt equation in the example given by Ritzema [Ref. 5] also gives F* = 1.0 m as height midway between the drains. The drain spacing is 66 m so table 1 gives the height of the water table midway between the drains at 33 m. Table 1 also shows the height of the water table at other distances from the drain, while the Hooghoudt equation only yields the midway height. The EnDrain software is more versatile.

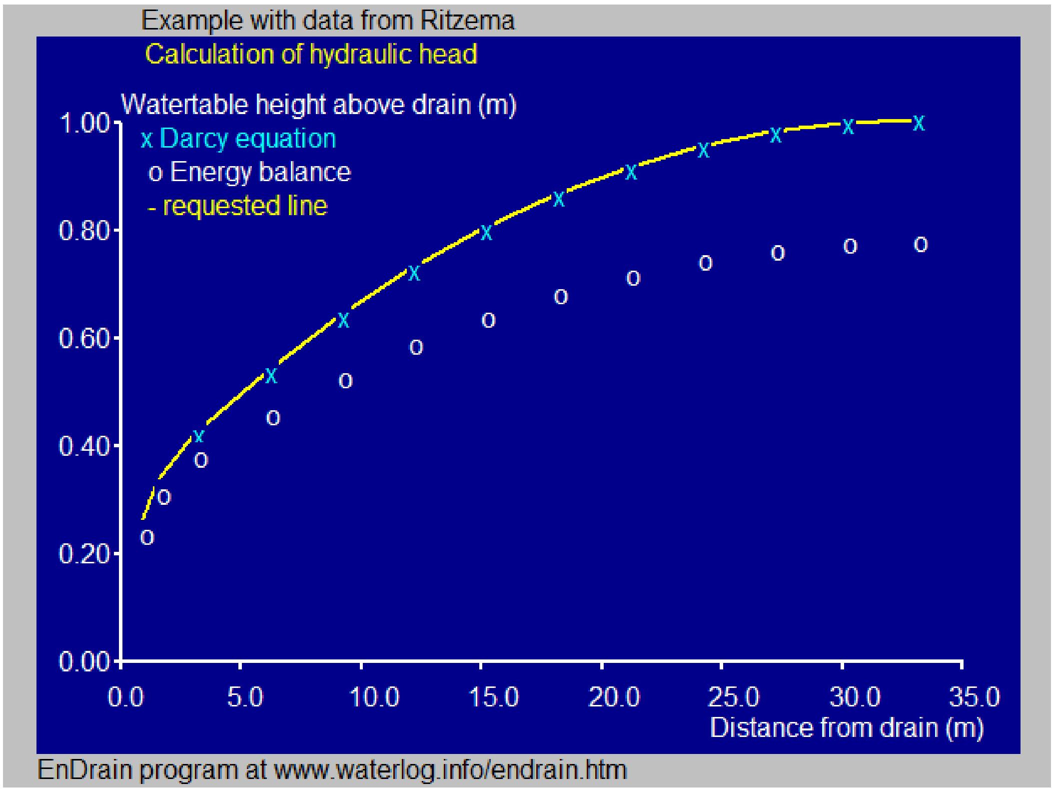

Fig. 4. The shape of the water table calculated with the energy balance equation and the Darcy equation (traditional) for the conditions given in the example. Graph produced by EnDrain. The Darcy equation

gives a higher water table as it ignores the incoming energy associated with the downward percolating water.

When a value of elevation Fn=1.0 m is accepted, the spacing can be considerably wider than 66 m so that the inclusion of the energy balance in the calculation of the drain spacing allows a cheaper drainage system.

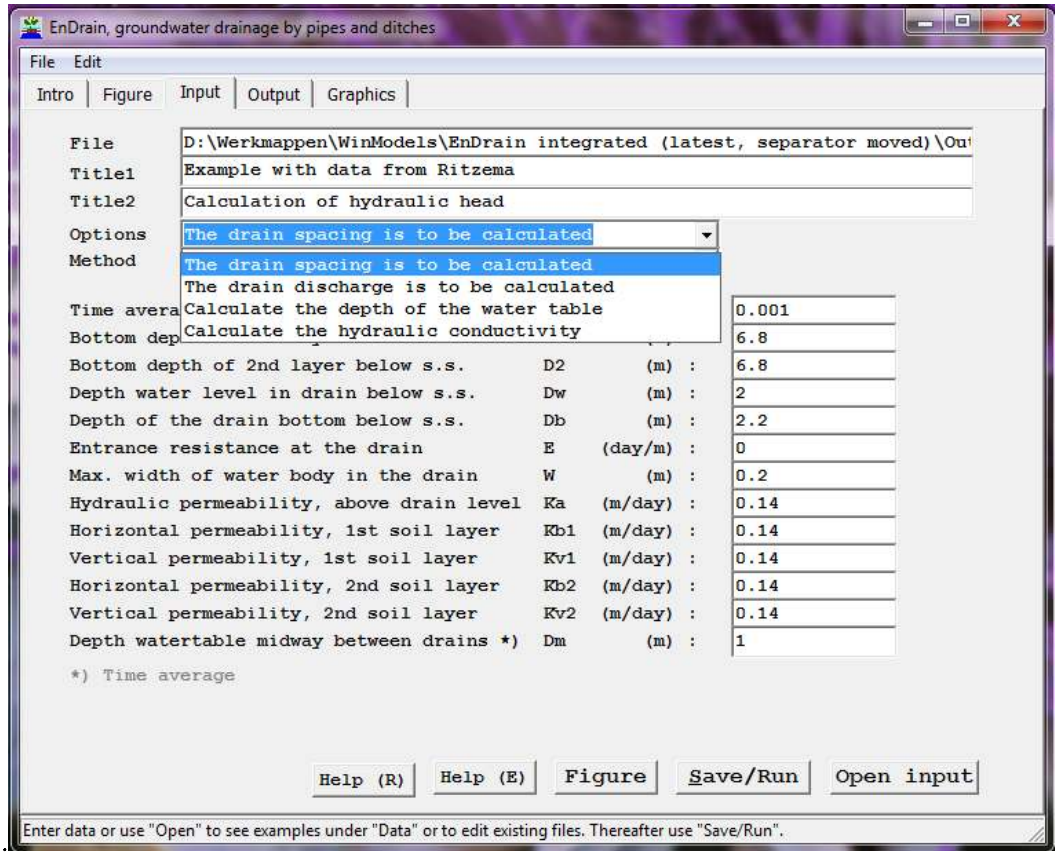

Fig. 5. Screen print off the EnDrain input tab sheet showing the data used in the previous example according to the data presented at the beginning of this section.

Figure 5 demonstrates the options in the EnDrain program like calculating the drain spacing, the drain discharge, the hydraulic head, or the hydraulic conductivity. The figure also clarifies that, in the example, no soil stratification and no anisotropic hydraulic conductivity were used.

5 Ditches

The principles of calculating the groundwater flow to ditches are similar as those to pipe drains.

When the width of the water body in the ditch (Wd) is twice its depth (Dd), then the principles are exactly the same (the ditches are neutral). Only the radius C of the drain must be replaced by an equivalent radius Ce = Dd = ½Wd (figure 6).

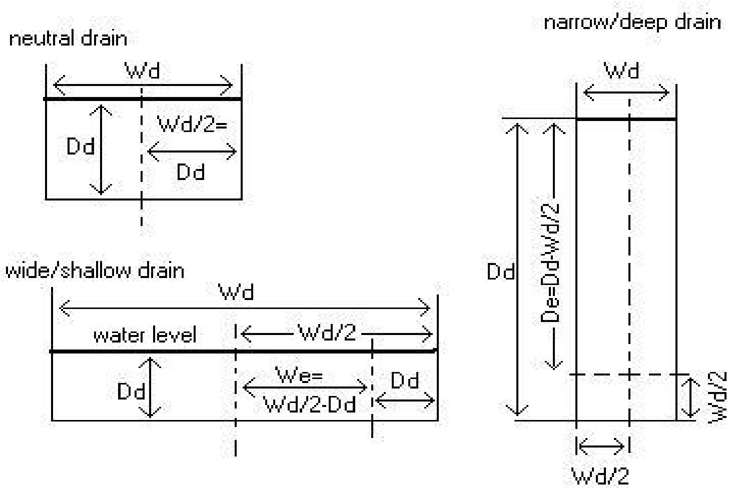

Fig. 6. Vertical and horizontal dimensions of ditch drains.

In conformity to the flow near pipe drains, the water enters the ditch from one side radially over a perimeter ½πCe. The numerical calculations start at the distance X = ½Wd from the central axis of the ditch. This means that the initial value i (Eq. 11) is changed into the integer value of:

$$i' = 1 + \tfrac{1}{2}Wd \,/\, U \qquad \text{(Eq. 14)}$$

The corresponding value of the horizontal distance X is indicated by X_i'.

The depth Y of the sloping impermeable layer is taken with respect to the water level in the drain. Otherwise the calculations are the same as for pipes.

For other situations (figure 6), we distinguish wide ditches (½Wd>Dd) from narrow ditches (½Wd<Dd).

For wide ditches, we replace the radius C by an equivalent radius Cw = Dd, and we define the excess width as We = ½Wd–Dd. The initial value i is again changed into i' as in equation 14. Further, the value $\underline{Y}_S$ in equation 7a changes into:

$$\underline{Y}_S' = \tfrac{1}{2}\pi X_S \qquad [\tfrac{1}{2}Wd < X_S < Xi'] \qquad \text{(Eq. 15)}$$

and the value of Z_S in Equation 9a changes into:

$$Z_S' = Kx(\underline{F}_S + \underline{Y}_S' + We) \qquad [\tfrac{1}{2}Wd < X_S < Xi'] \qquad \text{(Eq. 16)}$$

For narrow ditches, the radius C is replaced by an equivalent radius Cn = ½Wd, and we define the excess dept as De = Dd – ½Wd. Like before, the initial value i is changed into i'. Further, the factor Z_S in equation 9a is changed into:

$$Z_S'' = Kx(\underline{F}_S+\underline{Y}_S+De) \qquad [Dd<X_S<Xi'] \qquad \text{(Eq. 17)}$$

An example of results of calculations with the energy balance is given in table 2 for different ditches but otherwise with the same data as in the example for pipe drains. All ditches have a wet surface area of 2 m^2.

Table 2. Results of the calculations of the height Fn of the water table, taken with respect to the drainage level, midway between ditches of different shapes, using a numerical and iterative solution of the hydraulic energy balance for the the conditions described the example of Section 4, applying Equations 8 and 10 with steps U=0.05 m and making the adjustments as described in Section 5.

Width Wd (m)	Depth Dd (m)	Equivalen t radius (m)	Type of Ditch	Elevation of water table (Fn, m)
2	1	1	Neutral	0.55
3	2/3	2/3	Wide – shallow	0.52
1	2	½	Narrow – deep	0.52

From table 2 it is seen that the elevations Fn of the water table midway between the ditches are about 70% of the Fn value (0.76) calculated for pipe drains. Reasons are the larger equivalent radius, which reduces the contraction of and resistance to the the radial flow, and the larger surface width, which reduces the width of the catchment area.

6 Entrance Resistance

When entrance resistance [Ref. 7] is present, the water level just outside the drain is higher than inside by a difference Fe, the entrance head. It is calculated as Fe=R*2N*Er, where Er is the entance resistance (day/m). Now, the first value $\underline{F}_i$ of $\underline{F}_S$ is changed into $\underline{F}_i' = \underline{F}_i$+Fe. Otherwise the calculation procedure remains unchanged.

An example of the results of calculations with the energy balance for pipe drains with varying entrance heads, but otherwise with the same data as in the first example for pipe drains, is shown in Table 3. It is seen that the increment of elevation Fn with respect to Fe (Fn-Fe, column 4) decareases with increasing Er value. This means that part of the entance head loss is recovered further away from the drain thanks to a somewhat larger cross-section of the flow. Hence, the adverse effect of entrance head can be partly compensated

Table 3. Results of the calculations of the height Fn of the water table, taken with respect to the drainage level, midway between drain pipes, with different entrance heads, using a numerical and iterative solution of the hydraulic energy balance for the conditions described the example of Section 4, employing Equations 8 and 10 with steps U=0.05 m and making the adjustments as described in Section 6.

Entrance resistance (day/m)	Entrance head Fe (m) At the drain	Elevation of the water table Fn (m)	Fn – Fe
0.0	0.000	0.759	0.759
0.1	0.065	0.793	0.728
0.2	0.130	0.833	0.703
0.3	0.200	0.876	0.681
0.4	0.260	0.921	0.661
0.5	0.325	0.970	0.644

7 Anisotropy

The hydraulic conductivity of the soil may change with depth and be different in horizontal and vertical direction [Ref. 8]. We will distinguish a horizontal conductivity Ka of the soil above drainage level, and a horizontal and vertical conductivity Kb and Kv below drainage level. The following principles are only valid when Kv>R, otherwise the recharge R percolates downwards only partially and the assumed water balance Q= –R(N–X) is not applicable.

The effect of the conductivity Kv is taken into account by introducing the anisotropy ratio $A=\sqrt{(Kb/Kv)}$, as described by Boumans [Ref. 9].

The conductivity Kb is divided by this ratio, yielding a transformed conductivity: $Kt = Kb/A = \sqrt{(Kb.Kv)}$. As normally Kv<Kb, we find A>1 and Kt<Kb. On the other hand, the depth of the aquifer below the bottom level of the drain is multiplied with the ratio.

Hence the transformed depth is: Dt=A.D .

The distance $X_i = 2D/\pi$ (equation 5) of the radial flow now changes into $Xt = 2Dt/\pi$. When A>1, the transformed distance Xt is larger than Xi. The effect of the transformation is that the extended area of radial flow and the reduced conductivity Kt increase the resistance to the flow and enlarges the height of the water table.

Including the entrance resistance, the transmissivity Z_S (Equations 9a and 9b), for different types of drains, now becomes:

for pipe drains [$C<\underline{X}_S<Xt$]:

$$Z_S = \tfrac{1}{2}\pi Kt.\underline{X}_S + (Kb-Kt)Dd + Ka.\underline{F}_S$$

for neutral ditches [$Ce<\underline{X}_S<Xt$]:

$$Z_S = \tfrac{1}{2}\pi Kt.\underline{X}_S + (Kb-Kt)Dd + Ka.\underline{F}_S$$

for wide ditches [$Cw<\underline{X}_S<Xt$]:

$$Z_S = \tfrac{1}{2}\pi Kt.\underline{X}_S + (Kb-Kt)Dd + Kv.We + Ka.\underline{F}_S$$

for narrow ditches [$Cn<\underline{X}_S<Xt$]:

$$Z_S = \tfrac{1}{2}\pi Kt.\underline{X}_S - \tfrac{1}{2}Kt.Wd + Kb.Dd + Ka.\underline{F}_S$$

for all drains [$Xt<\underline{X}_S<N$]:

$$Z_S = Kt.Dt + Ka.\underline{F}_S$$

The suggestion of Boumans [Ref. 9] to use the wet perimeter of the ditches to find the equivalent radius, without making a distinction between wide and narrow drains, is not followed as this would lead to erroneous results for narrow and very deep drains, especially when they penetrate to the impermeable layer. In the latter case there is no radial flow but the use of the wet perimeter would introduce it. The proposed method does not.

Table 4 gives an example of energy balance calculations for pipe drains in soils with anisotropic hydraulic conductivity using Ka = Kb = 0.14, as in the previous examples, and Kv = 0.14, 0.014 and 0.0014. This yields anisotropy ratios A = 1, 3.16, and 10 respectively. All other data are the same as in the previous examples.

The table shows that the height Fn increases with increasing ratio A and the increase is higher for the smaller pipe drains than for the larger ditches. This is due to the more pronounced contraction of the flow to the pipe drains than to the ditches and the associated extra resistance to flow caused by the reduction of the hydraulic conductivity for radial flow from Kb to Kt. The narrow/deep ditches show by far the smallest increase of the height Fn, due to their deeper penetration into the soil by which they make use of the higher horizontal conductivity Kb.

Unfortunately, it is practically very difficult to establish and maintain such deep drains at field level.

When the height Fn would be fixed, one would see that the spacing in anisotropic soils is by far the largest for the narrow and deep ditches. Neutral drains would have smaller spacing than wide drains, i.e. the advantage of wide ditches in isotropic soils vanishes in anisotropic soils. The pipe drains would have the smallest spacing.

8 Layered (an)isotropic soils

The soil may consist of distinct (an)isotropic layers. In the following model, three layers are discerned.

The first layer reaches to a depth D1 below the soil surface, corresponding to the depth Wd of the water level in the drain, and it has an isotropic hydraulic conductivity Ka. The layer represents the soil conditions above drainage level.

The second layer has a reaches to depth D2 below the soil surface (D2>D1). It has horizontal and vertical hydraulic conductivities K2x and K2v respectively with an anisotropy ratio A2.The transformed conductivity is Kt2 = K2x/A2.

The third layer rests on the impermeable base at a depth D3 (D3>D2). It has a thickness T3=D3–D2 and horizontal and vertical hydraulic conductivity Kx3 and Kv3 respectively with an anisotropy ratio A3. The transformed conductivity is Kt3= K3x/A3, and the transformed thickness is Tt3=A3.T3

When the thickness T3 = 0 and/or the conductivity K3 = 0 (i.e. the third layer has zero transmissivity and is an impermeable base), the depth D2 may be both larger or smaller than the bottom depth Db of the drain. Otherwise, the depth D2 must be greater than the sum of bottom depth and the (equivalent) radius (C* = C, Ce, Cw, or Cn) of the drain, lest the radial flow component to the drain is difficult to calculate.

For pipe drains, neutral and wide ditch drains, and with D2 > Dw + C* = Dw + Dd, the transformed thickness of the second soil layer below drainage level becomes

$$Tt2 = A2(D2-Dw).$$

For narrow ditches we have similarly:

$$Tt2 = A2(D2 - Dw - \tfrac{1}{2}Wd + Dd)$$

With the introduction of an additional soil layer, the expressions of transmissivity $\underline{Z}_S$ in Section 7 need again adjustment, as there may two distances Xt (Xt1 and Xt2) of radial flow instead of one, as the radial flow may occur in the second and the third soil layer:

$$Xt1 = 2T2t/\pi$$

$$Xt2 = Xt1 + 2Tt3/\pi$$

With these boundaries, the transmissivities become

for pipe drains [C<$\underline{X}_S$<Xt1]:

$$Z_S = \tfrac{1}{2}\pi Kt2.\underline{X}_S + (Kx2-Kt2)Dd + Ka.\underline{F}_S$$

for neutral ditches [Ce<$\underline{X}_S$<Xt1]:

$$Z_S = \tfrac{1}{2}\pi\, Kt2.\underline{X}_S + (Kx2-Kt2)Dd + Ka.\underline{F}_S$$

for wide ditches [Cw<$\underline{X}_S$<Xt1]:

$$Z_S = \tfrac{1}{2}\pi\, t2.\underline{X}_S + (Kx2-Kt2)Dd + Kv2.We + Ka.\underline{F}_S$$

and:

for narrow ditches [$Cn<\underline{X}_S<Xt$]:

$$Z_S = \tfrac{1}{2}\pi\, Kt2.\underline{X}_S - \tfrac{1}{2}Kt2.Wd + Kx2.Dd + Ka.\underline{F}_S$$

for all drains:

if [$Xt1<\underline{X}_S<Xt2$]:

$$Z_S = Kt2.Tt2 + \tfrac{1}{2}\pi\, Kt3.\underline{X}_S + Ka.\underline{F}_S$$

if [$\underline{X}_S>Tt2+Tt3$]:

$$Z_S = Kt2.Tt2 + Kt3.Tt3 + Ka.\underline{F}_S$$

Table 4. Results of the calculations of the height Fn (m) of the water table, taken with respect to the drainage level , midway between pipe drains and ditches in anisotropic soils with a fixed value of the horizontal hydraulic conductivity Kb=0.14 m/day, using a numerical and iterative solution of the hydraulic energy balance for the conditions described the previous examples, employing Equations 8 and 10 with steps U=0.01 m and making the adjustments as described in Section 7.

Vertical hydraulic conductivity Kv (m/day)	Height Fn of the water table above drain level (m)			
	Pipe drains C = 0.1 m	Ditches		
		Neutral Wd = 2 m Dd = 1 m	Wide Wd = 3 m Dd = 2/3 m	Narrow Wd = 1 m Dd = 2 m
0.14	0.76	0.55	0.52	0.52
0.014	1.13	0.69	0.73	0.59
0.0014	1.63	1.00	1.11	0.74

An example will be given for pipe drains situated at different depths within the relatively slowly permeable second layer having different anisotropy ratios and being underlain by an isotropic, relatively rapidly permeable, third layer with different conductivities. We have the following data:

N = 38 m   C = 0.05 m   R = 0.007 m/day
D1 = 1.0 m   D2 = 2.0 m   D3 = 6.0 m
Kx2 = 0.5 m/day   Kx3 = 1.0 m/day
Ka = 0.5 m/day Kv2= 0.5 m/day   Kv3 =1.0 m/day
and variations:
Kv2 = 0.1 m/day   Kv2 = 0.05 m/day
Kx3 = Kv3 = 2.0 m/day   Kx3 = Kv3 = 5.0 m/day

The results are shown in Table 5.

Table 5. Results of the calculations of the height Fn (m) of the water table, taken with respect to the drainage level, midway between pipe drains in a layered soil of which the second layer, in which the drains are situated, has varying anisotropy ratios with a fixed value of the horizontal hydraulic conductivity Kx2=0.5 m/day, using a numerical and iterative solution of the hydraulic energy balance for the conditions described the example of Section 8, employing Equations 8 and 10 with steps U=0.05 m and making the adjustments as described in Section 8.

Hydr. Cond. 3rd layer Kx3=Kv3 (m/day)	Vert. Hydr. Cond. 2nd layer Kv2 (m/day)	Anisotropy ratio A2 2 nd layer (−)	Height Fn of the water table above drainage level (m)
1.0	0.5	1.0	0.54
1.0	0.1	2.24	0.75
1.0	0.05	3.13	0.86
2.0	0.5	1.0	0.45
2.0	0.1	2.24	0.67
2.0	0.05	3.13	0.79
5.0	0.5	1.0	0.37
5.0	0.1	2.24	0.60
5.0	0.05	3.13	0.74

These results indicate that both the conductivity of the 3rd layer and the anisotropy of the 2nd layer, in which the drains are situated, exert a considerable influence on the height Fn.

In the Netherlands, it is customary to prescribe a minimum permissible depth of the water table of 0.5 m at a discharge of 7 mm/day, which is exceeded on average only once a year. In the example, with a drain depth of 1.0 m, this condition is fulfilled when the height Fn is at most 0.5 m. Here, this occurs when Kv2 is at least 0.5 m/day and when Kx3 = Kv3 is at least 2.0 m. To meet the prescription in the other cases of the example, either the drain depth should be deeper or the drain spacing narrower.

9 Equivalent hydraulic conductivity

For ditch drains reaching the impermeable layer or pipe drains resting on the impermeable layer, Hooghoudt derived the following equation [Ref. 5]:

$$Q = \{8K_b\, D\, F_r + 4K_a\, F_r^2\} / L^2$$

For the meaning of the symbols see figures 2 and 5. Note that J_0 in figure 1 equals D in figure. 5. Q is the drain discharge in m^3/day per m^2 surface area drained or m/day. This equation is based on the partial energy balance,

With a numerical approach, the discharge-weighted average transmissivity (Z_{av}) can be calculated. It is found by dividing the the distance from the drain to the point midway between the drains into small steps and determining in each corresponding vertical section the transmissivity below the water level in the well (in m^2/day) and the discharge (in m^2/day). Next these two quantities are multiplied and the products are added. Finally Z_{av} is obtained by dividing this sum of products by the sum of the discharges.

Using $K_e = Z_{av}/D$, where K_e is the equivalent hydraulic conductivity, one will be able to use the above well flow equation for fully penetrating drains to a situation with partially penetrating drains not reaching the impermeable layer replacing here K_b by K_e.

10 Conclusions

Application of the complete energy balance of groundwater flow to pipe and ditch drains leads to lower elevations of the water table or, if the elevation is fixed, to a wider drain spacing compared to standard formulas A numerical solution, moreover, can give the shape of the water table.

Further, it can take entrance resistance and anisotropy of the soil's hydraulic conductivity into account.

Calculations with the full energy balance need be done on a computer because of the cumbersome iterative, numerical procedure required. EnDrain may be useful software for that purpose. A similar program for drainage by pumped well is also available [Ref. 10].

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